OPTIMAL TIME AND ENERGY EFFICIENCY IN LEGGED ROBOTICS

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ABSTRACT

Although bio-inspired legged robots have advantageous mobility, they can be very inefficient. Their intrinsic walking mobility is sometimes outweighed by the inefficiency of their drive-train. Some of these inefficiencies are due to collision losses, but they are also due to suboptimal powering schemes. This paper addresses the powering schemes and seeks to clearly delineate an optimal solution to powering the walking motion of a two-legged or biped walker. We examine a simplified model of locomotion called the "rocket car" to extract the meaningful parameters that affect time and energy cost. Using Pontryagin's Maximum Principle, we dissect the cost function, the state equation, co-state equation, and control input constraints to describe the optimal control. The result of the paper shows a "bang-off" control, and we describe the "coasting line" between these extremes. It is not possible to find a complete closed-form solution for the problem, and numerical methods, such as dynamic programming must be used for future simulation and visualization of the results.

INTRODUCTION

Legged Robotics has long held the promise of mobility [1]. Until recently, most research in legged robotics has focused on stability. Recent research [2] has come to the conclusion that stability is a necessary, but not sufficient condition. Efficiency is a key factor that must not be overlooked. Particularly, we examine efficiency in the powering scheme of a legged robot.

As a rough order of magnitude (ROM) estimate, we approximate the powering scheme for a legged system with the so-called "rocket car" problem [3]. This deliberately simplifies the problem by leaving out potential energy changes in the system. We will adjust the problem to reach the origin with maximum velocity, instead of zero velocity, to simulate the transport of mass as quickly as possible through a single stride.

Poyntryagin's Maximum Principle (PMP) will be used to study the control scheme, which is the foundation of modern control theory and has its roots in the solutions of problems required for space technology (hence the name "rocket car"). This is an arcane mathematical principle, which is best understood from an economics point of view. The interested reader is directed to a few lucid treatments in [4-5].

POSITIONING PROBLEM WITH A TIME AND KINETIC ENERGY COST FUNCTION ϕ

In the textbook treatment of the rocket car problem problem, the cost function, ϕ, is assumed to be for a time optimal problem (TOP), fuel, energy, or a mixed case of these. The standard treatment for the energy cost function is $\phi = u^2$, which turns out to be the case if u is a current source. In this paper, we derive a treatment for energy where u is a control force instead. We show that the cost function for energy is $\phi = uv$.

Given:

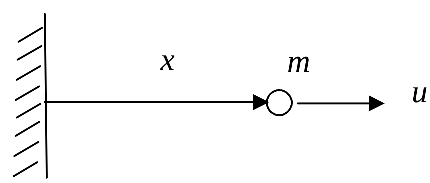

Figure 1: Positioning Problem

Required:

Drive the mass back to the origin at $x = 0$ with maximum velocity, while minimizing some integral-type cost, J, where:

$$J = \int_0^T \phi(\mathbf{x}, u)\, dt \tag{1}$$

In this problem, we choose two components for the cost (time and energy)

$$J = \int dt + \int dE \tag{2}$$

Substituting for power

$$J = \int dt + \int \frac{dE}{dt} dt \tag{3}$$

Substituting for kinetic energy

$$\int \frac{dE}{dt}dt = \int \frac{d}{dt}\left(\frac{1}{2}mv^2\right)dt \qquad (4)$$

Noting that $m\dot{v} = ma = u$

$$\int mv\dot{v}dt = \int uvdt \qquad (5)$$

Finally, substitute the above result into (1) to yield

$$J = \int_0^T (k + uv)\,dt \qquad (6)$$

Where k is a positive constant. Thus we have two components for the cost, with a constant k to measure the relative importance between the two.

POYNTRYAGIN'S MAXIMUM PRINCIPLE (PMP)

1st step in the Poyntryagin Maximum Principle: Treat cost as an additional state. The state equations are thus given by:

$$\dot{\mathbf{x}} = \begin{pmatrix} \dot{x}_0 \\ \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} k+uv \\ v \\ u \end{pmatrix} = \begin{pmatrix} k+ux_2 \\ x_2 \\ u \end{pmatrix} \qquad (7)$$

Since we want to approach the origin with maximum velocity, u is bounded by zero [6].

$$-1 \le u \le 0 \qquad (8)$$

In other words, while you may want to back off the gas, you would never want to apply the brakes.

2nd step in the Poyntryagin Maximum Principle: We introduce the co-state variables to form the Hamiltonian:

$$H = z_0\dot{x}_0 + z_1\dot{x}_1 + z_2\dot{x}_2 \qquad (9)$$

Substituting (7) we see that

$$H = z_0\left(k + ux_2\right) + z_1 x_2 + z_2 u \qquad (10)$$

The co-state equations are prescribed by Hamilton's equations:

$$\dot{z}_0 = -\frac{\partial H}{\partial x_0} = 0 \qquad (11)$$

$$\dot{z}_1 = -\frac{\partial H}{\partial x_1} = 0 \qquad (12)$$

$$\dot{z}_2 = -\frac{\partial H}{\partial x_2} = -z_1 - z_0 u \qquad (13)$$

3rd step in the Poyntryagin Maximum Principle

Solve the above co-state equations. The equation for $\dot{z}_0$ shows that $z_0 = const$, and the PMP requires that this constant should be *negative*. Without loss of generality we can choose $z_0 = -1$. The two solutions to (11) and (12) are thus:

$$z_0 = -1 \qquad z_1 = A \qquad (14)$$

The solution to the last equation (13) is given by substituting (14), which yields $\dot{z}_2 = u - A$, and the solution becomes

$$z_2 = \int udt - At + B \qquad (15)$$

We've previously shown $\int udt = mv - mv_0$, so that

$$z_2 = mx_2 + B - mv_0 - At \qquad (16)$$

4th step in the Poyntryagin Maximum Principle: Find the supremum of H as a function of u. The Hamiltonian is given by substituting (14), (16), and (7) into (10):

$$H = -k - ux_2 + Ax_2 + u\left(mx_2 + B - mv_0 - At\right) \qquad (17)$$

To maximize H as a function of u, we must maximize the term uq where

$$q = \left(m-1\right)x_2 + B - mv_0 - At \qquad (18)$$

If $m = 1$ then $q' = B' - At$ where $B' = B - mv_0$ and since q' is a linear function of t there is at most one zero crossing for q'. Thus

$$u = \begin{vmatrix} -1 & if & q' < 0 \\ 0 & if & q' > 0 \end{vmatrix} \qquad (19)$$

PMP has thus shown that we have "bang-off" control with a single switch. This makes intuitive sense, since we would want to apply the force early on in order to gain the time savings accrued throughout the stride, but we would want to shut the force off after some time period in order to save energy.

OPTIMAL COST

We desire to find the optimal cost J^*. We assume the optimal solution is "bang-off" as described in the previous section. First, we proceed with a bang, i.e., from the initial position X to the switching position s we apply full reverse force ($u=-1$), and then at s, we cut the engines off ($u=0$) and coast through the origin with a cruising speed of "a." as shown in the phase plot in Figure 1.

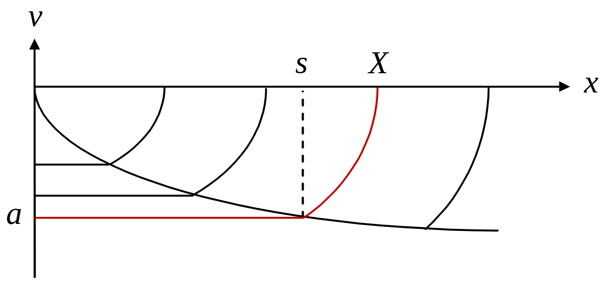

Figure 2: The Switching Curve

We can rewrite the cost integral (6) as a function of x instead of t:

$$J=\int_0^T (k+uv)\,dt=\int_{x_1}^{x_2} k\frac{dx}{v}+\int_{x_1}^{x_2} udx \qquad (20)$$

where we have used the definition of velocity $v=\frac{dx}{dt}$ and the definition of work $E=\int F\cdot ds=\int udx$.

Working backwards from the initial position X at zero velocity until the switch at position s and velocity a, the time cost becomes:

$$\int_X^s k\frac{dx}{v}=km\int_0^a \frac{dv}{u}=km\int_0^a \frac{dv}{-1}=-kma \qquad (21)$$

which uses the physics result $ads=vdv$ recast as $udx=mvdv$.

The energy cost for this period is given by:

$$\int_X^s udx=m\int_0^a vdv=\frac{ma^2}{2} \qquad (22)$$

There is only time cost as we cruise at velocity a from position s to the origin:

$$k\int_s^0 \frac{dx}{a}=-\frac{ks}{a} \qquad (23)$$

Putting together the above three equations we get the optimal cost:

$$J^*=-kma+m\frac{a^2}{2}-k\frac{s}{a} \qquad (24)$$

This will always be positive since a is negative.

We really would like a result that is only in terms of the initial conditions. So we seek a formula that relates s to a. Integrating $udx=mvdv$:

$$\int_X^s dx=\int_0^a \frac{mvdv}{-1} \qquad (25)$$

We find the relation:

$$s=X-\frac{ma^2}{2} \qquad (26)$$

Substituting the above result for s into (24):

$$J^*=-kma+m\frac{a^2}{2}-\frac{k}{a}\left(X-\frac{ma^2}{2}\right) \qquad (27)$$

Simplifying:

$$J^*=-kma+m\frac{a^2}{2}-\frac{k}{a}X+\frac{kma}{2}$$
$$=-\frac{kma}{2}+m\frac{a^2}{2}-\frac{kX}{a} \qquad (28)$$

We desire a result that minimizes $J(a)$:

$$\frac{dJ^*}{da}=0=-\frac{km}{2}+ma+\frac{kX}{a^2} \qquad (29)$$

We can check this result using implicit differentiation of (24)

$$\frac{dJ^*}{da}=0=-km+ma-k\left(\frac{s'}{a}-\frac{s}{a^2}\right) \qquad (30)$$

Which we substitute into (30) along with $s'=-ma$:

$$0=-km+ma-k\left[-m-\left(\frac{X}{a^2}-\frac{m}{2}\right)\right] \qquad (31)$$

Simplifying:

$$0 = -k\cancel{m} + ma + k\cancel{m} + k\left(\frac{X}{a^2} - \frac{m}{2}\right)$$
$$0 = -\frac{km}{2} + ma + \frac{kX}{a^2} \qquad (32)$$

which is the same as (29)

This result yields the following cubic equation for a:

$$ma^3 - \frac{km}{2}a^2 + kX = 0 \qquad (33)$$

This equation must be solved numerically to describe the "coasting line" shown in Figure 2, in terms of the initial conditions of the problem.

CONCLUSIONS AND FUTURE WORK

Describing the "coasting line" only gives an initial insight into the problem. This must be followed by numerical analysis to check these results, and verify the theory. Other models including friction, position uncertainty, and disturbance would add to the richness of the problem and increase its difficulty. The authors wish to thank Grant Gerhart and Rob Karlsen for funding this research through the In-Lab Innovative Research (ILIR) program at US Army RDECOM/TARDEC.

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