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DEVELOPMENT OF SOFT SOIL MODELS USING THE DISCRETE ELEMENT METHOD (DEM) FOR TWO-WAY ALTAIR EDEM + MBD OFF-ROAD MOBILITY SIMULATIONS

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ABSTRACT

A numerical modelling methodology for the in-situ axial pressure-displacement response of soft soils is developed based on the Discrete Element Method. The resulting models achieve good agreement with the idealized Beirnstein form of the 'p-z' equation describing soft soil uniaxial pressure-displacement response. A method of determination of appropriate input parameters for the models from bevameter measurements of soft soils is proposed.

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1. INTRODUCTION

The Discrete Element Method (DEM), a numerical method for computing the motion of large numbers of particles, is increasingly used to model complex terramechanics. Advancements in hardware and software have led to the development of large scale coupled DEM - Multi-Body Dynamics (MBD) simulations capable of capturing the complex interactions between wheeled and tracked vehicles and soft soils.

NATO activity ATV-248 [1] is actively developing a Next Generation NATO Reference Mobility Model (NG-NRMM) and identifies modelling complex terramechanics for capturing soft soil responses as one of the primary research goals.

The modelling of complex terramechanics is important across a range of industries. Example applications include military and civilian off-road vehicles, heavy equipment operating in the off-road environment, agricultural equipment, and extra-planetary rovers. Edwards, 2018 [2] showed how coupled Altair EDEM™ + MBD simulations have been deployed for predicting mobility of tracked and wheeled vehicles in soft soil conditions and proposed a testing procedure for calibrating DEM soil properties using Bekker-Wong [3] [4] parameters to produce a database of DEM soil models.

Traditional Bekker-type terramechanics methods do not consider the soil profile, soil dynamics, or transient wheel dynamics [5]. The 'dynamic Bekker' method is used to overcome these limitations allowing for its deployment in MBD simulations.

To date, soil models used in coupled EDEM + MBD simulations have not specifically been calibrated in line with traditional methods for characterizing the response of soft soils pioneered by M. G. Bekker [3] and J. Y. Wong [4] making comparison to measured field data and existing analytical terramechanics methods challenging.

To relate simple terremachanics models to DEM-based complex terramechanics models it is desirable that any DEM-based response library for modelling complex terramechanics should be related to traditional Bekker-type models via the publishing of full or reduced Bekker-Wong parameters. A DEM-based soils response library should qualitatively and quantitatively capture the mechanics of physical soil responses and be easily related to a diverse set of soil types.

In this paper traditional methods for characterizing soils are presented. The NG-NRMM is discussed in relation to its stated research goal for modelling complex terramechanics. An overview of the DEM is given and a meso-scopic modelling approach for soils, utilizing the Edinburgh Elastic Plastic Adhesion (EEPA) contact model, is described. To capture the in-situ uniaxial pressure-displacement response of soft soils a methodology for reproducing the uniaxial bevameter test which is applied in the published NG-NRMM Cooperative Demonstration of Technology (CDT) event data is developed. The numerical model input parameters are modified to reproduce the in-situ uniaxial pressure-displacement response of soils with a wide range of physical properties. The methodology and the challenges associated with achieving a high degree of accuracy for this response are discussed.

2. CHARACTERIZING SOILS

The Unified Soil Classification System (USCS) is a soil classification system used for engineering purposes and is based on laboratory determination of particle-size characteristics, liquid limit, and plasticity index [6]. The classification system is represented by a two-letter symbol detailed in Table 1. The USCS does not contain information on soil macro-mechanical characteristics but it is a convenient method for broadly classifying commonly occurring soils.

First Letter		**Second Letter**	
G	Gravel	P	Poorly graded
S	Sand	W	Well graded
M	Silt	H	High plasticity
C	Clay	L	Low plasticity
O	Organic		

Table 1. USCU soil classification system

Characterization of soils for the purpose of high fidelity, complex terramechanics computational modelling is an area of active research. Soft soil characterization exercises rely on reproduction and comparison of results against physical testing. These have included cone penetrometer and unconfined compression testing [7], direct shear, pressure-displacement and constant slip ratio wheel tests [5] and triaxial compression tests [8]. These tests vary in complexity for both the collection of physical measurement data and reproduction in DEM simulations.

The cone penetrometer test is a practical method for soil characterization due to the simplicity of operation. McCullough, 2017 [9] notes that a cone penetrometer is not a very close analog to vehicle running gear bearing and tractive load interactions with soil. One limitation of the cone penetrometer test for soil characterization in DEM simulations is that the computational cost of modelling the full scale test is prohibitive, leading to the necessity of using numerical particles, several orders of magnitude larger than the physical size. This results in the loss of micro-scale fidelity of the model.

Independent measurement of soil pressure-displacement response and response due to shear loading have emerged as promising methods for characterizing soils [5] for complex terramechanics computational modelling. Measurement of these responses using a bevameter form the basis of most modern simple terramechanics models [9].

2.1. Normal response

Soil normal response is represented by ‘p-z’ equations where p is normal bevameter pressure and z is displacement. Several forms of p-z equations exist.

Where k [N/m^{n+2}] and n are best fit parameters, Bernstein, 1913 [10] originally proposed a power law form of the plastic limit pressure:

$$p = kz^n \qquad (1)$$

Bekker, 1969 [3] proposed the introduction of a running gear dimension, b. Where k_c [N/m^{n+1}] and k_φ [N/m^{n+2}] are the cohesive modulus of sinkage and frictional modulus of sinkage respectively. The running gear dimension, b, is typically the width or radius of a circular contact area

$$p = \left(\frac{k_c}{b} + k_\varphi\right) z^n \qquad (2)$$

Recording pressure-displacement response for two different bevameter diameters it is possible to solve for k_c and k_φ. The effect of independent parameter variation for the Bekker p-z equation is show in Figure 1.

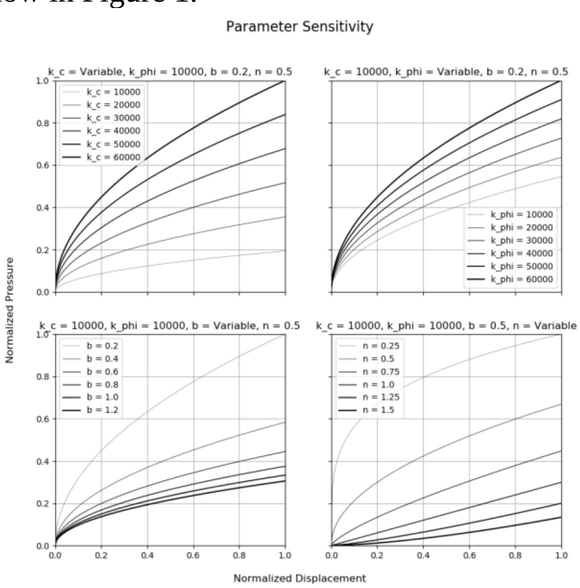

Figure 1. Normalized pressure-displacement response showing the effect of independent parameter variation

Reese, 1964 [11] described (2) as inadequate, that it fits sand quite well but that it does not suit cohesive soils and that the equation is dimensionally inconvenient and proposed a p-z equation of the form;

$$p = \left(ck_c{'} + \gamma \frac{b}{2} k_\varphi{'}\right)\left(\frac{z}{b}\right)^n \quad (3)$$

Where c is cohesion [N/m^2], γ is soil density [N/m^3] and k_c’ and k_φ’ are the dimensionless soil cohesive and friction moduli of sinkage.

Equations (2) and (3) both introduce a running gear dimension, b, to augment for real world running gear dimensions. A common practice is to use a circular bearing for loading an unconfined or an in-situ soil sample for recording the pressure-displacement response. Meyerhof, 1961 [12] points out that the relationship between the pressure-displacement response and plate shape is complex and varies greatly with depth and soil type.

2.2. Shear response

Janosi and Hanamoto, 1961 [13] propose a ‘τ-j’ shear stress-shear displacement relationship of the form

$$\tau = (c + \sigma tan\varphi)\left(1 - e^{-\frac{K}{j}}\right) \qquad (4)$$

Where σ is normal stress [N/m^2], φ is the angle of internal friction [deg] and K is the soil internal deformation modulus [m].

3. NEXT GENERATION-NATO REFERENCE MOBILITY MODEL (NG-NRMM)

NATO’s Applied Vehicle Technology (AVT) Panel formed Research Task Group AVT-248 to develop a NG-NRMM. By leveraging advanced physics models and modern computing a NG-NRMM would generate improved predictive capabilities for the mobility of ground platforms over a wide range of terrains. The primary focus of the upgrade of the NRMM to a NG-NRMM is to leverage technological advances in computational capacity and simulation software [14].

A Cooperative Demonstration of Technology (CDT) event was held in September 2018 at The Keweenaw Research Center (KRC) where a prototype process of loosely integrated technologies and tools contributed by committee members and software developers was demonstrated [15]. Modelling complex terramechanics for capturing soft soil responses is one of the primary research goals outlined by AVT-248 for development of a NG-NRMM to overcome the limitations of existing models [5]. NG-NRMM complex terramechanics models are those that, given any 3D soil loading condition by a vehicle surface, can accurately predict the 3D reaction forces on the vehicle surface and the 3D soil flow/deformation including permanent deformation [16].

Different macro-scale models for complex terramechanics where soil particles are lumped to form a virtual particle or finite element were investigated as part the complex terramechanics research. These included Lagrangian and Eularian finite element (FE) based methods and DEM, Smoothed Particle Hydrodynamics (SPH), Material Point Method (MPM) and Particle Finite Element Method (PFEM) particle based methods. The DEM scored highest in an exercise identifying complex terramechanics model technology readiness where each method was scored against measures for accuracy/generality of soil material models, range of soil deformation, ability to include embedded obstacles, fidelity of the soil-vehicle interface, computational speed, experimental validation and their current use in vehicle mobility [16].

3.1. In-situ field data

The NG-NRMM CDT published bevameter pressure-displacement, shear ring response and cone penetrometer data from in-situ measurements on several soil types at The Keweenaw Research Center. The NG-NRMM bevameter setup is illustrated in Figure 2.

Normal pressure-displacement response was measured using circular bevameters with 4 inch (0.1016 m) and 6 inch (0.1524 m) diameters. Raw data for a subset of results which were recommended by the KRC to be good representative model fits [17] are presented in Figures 3 to 6. Normal Bekker-Wong parameters k_c, k_φ and n are derived from the pressure-displacement raw data is presented in Table 2.

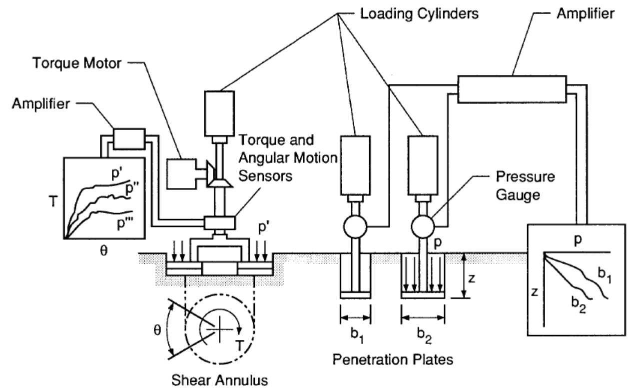

Figure 2. NG-NRMM bevameter

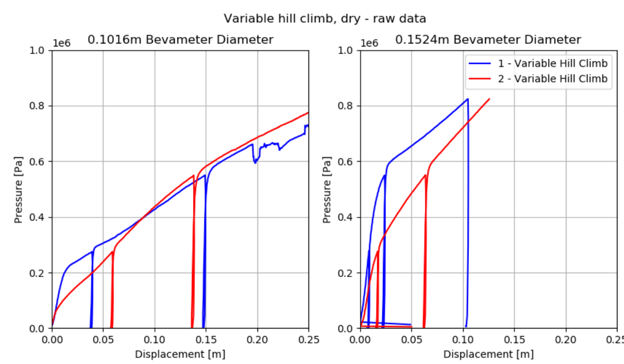

Figure 3. Variable hill climb, dry - raw data

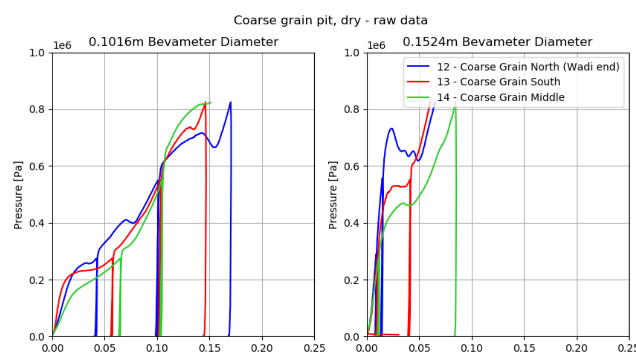

Figure 4. Coarse grain pit, dry - raw data

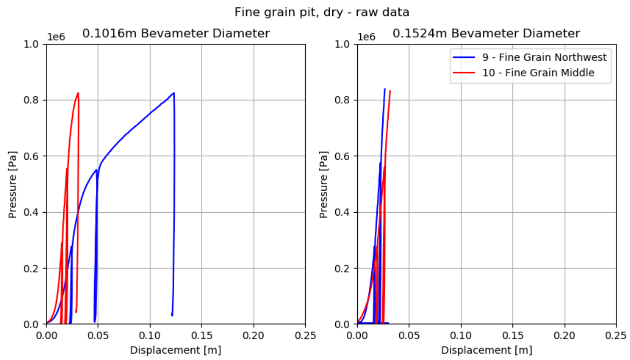

Figure 5. Fine grain pit, dry - raw data

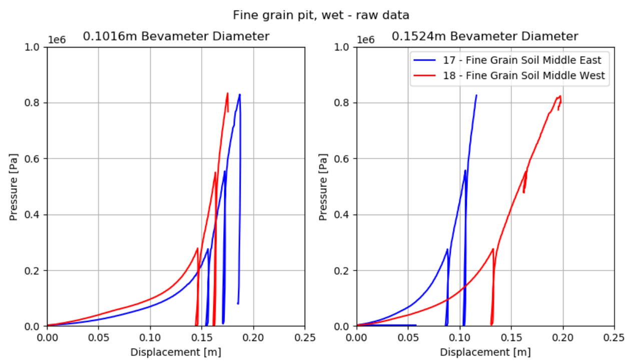

Figure 6. Fine grain pit, wet - raw data

Location	n	k_c [kN/m^{n+1}]	k_φ [kN/m^{n+2}]
Variable hill climb, 2NS sand, dry	0.5	56.2	410.8
Fine grain pit, dry	1.8	58125.4	-580375.0
Coarse grain pit, dry	0.6	74.0	1087.3
Fine grain pit, wet	3.3	-31513.1	81614.6

Table 2. NG-NRMM CDT normal Bekker-Wong bevameter parameters

3.2. Laboratory data

The NG-NRMM CDT published a set of laboratory data for soils sampled at a range of locations at the KRC. Particle size, direct shear, triaxial and pressure cell compression analysis', among others, were carried out.

4. DISCRETE ELEMENT METHOD

The Discrete Element Method is a particle scale numerical method for modelling particulate materials, in which particle motion is computed by an explicit integration of Newtons equations of motion [18]. The interaction forces between particles are described via analytical contact models, on the basis of small overlaps between particle and geometry elements. A classical example is the Hertz-Mindlin contact model based on Hertzian contact theory [19] extended by Mindlin [20] to include tangential forces.

The DEM has been extended to provide co-simulation capability with other widely used Computer Aided Engineering (CAE) simulation methods including Finite Element Analysis (FEA) [21], Computational Fluid Dynamics (CFD) [22] and Multi-Body Dynamics [23].

Industries where the DEM has been applied for modelling large, complex systems of particles include mining, heavy equipment, and pharmaceutical and process industries. Recently it has been demonstrated that EDEM can be used to model soft soils in fully coupled two way DEM + MBD simulations of wheeled and tracked vehicles.

The use of the DEM for modelling complex terramechanics applications is advantageous due to its ability to capture the macro-mechanics of particulate solids under both quasi-static and highly dynamic responses, commonly observed in real soft soils. Research shows that the DEM can be used to accurately model the mechanics of soft soils. Mustafa, 2015 [24] demonstrated the use of DEM for modelling soil-tool interaction for agricultural tillage applications. Janda, 2015 [7] showed how using a visco-elasto-plastic frictional adhesive contact model it is possible to capture the macro-mechancial behaviour of cohesive materials while Zuh, 2008 [25] pointed out that soil particle arrangement, or packing, has a significant impact on macroscopic soil behavior.

4.1. Edinburgh elastic plastic adhesion (EEPA) contact model

Selection of a DEM contact model for modelling soft soils for use in large scale off-road mobility applications is dependent primarily on two factors. Firstly, the contact model must be able to capture the macro-mechanical behavior observed in physical soft soils - namely hysteretic and cohesion effects and secondly, the model must be computationally efficient so that it may be practically applied in full vehicle scale simulations. The EEPA contact model is a promising model for accurately capturing soft soil response for these reasons.

The EEPA contact model is capable of capturing the complex visco-elastic-hypo-plastic behaviour of cohesive soils while utilizing the computationally efficient meso-scopic modelling approach whereby particulate materials are modelled using numerical particles of an intermediate scale between the physical particle scale and the scale of the system of interest [26]. This enables the numerical models to predict the history stress dependent macro-mechanical response of soils, while allowing for practical solve times for simulating soil beds of significant size.

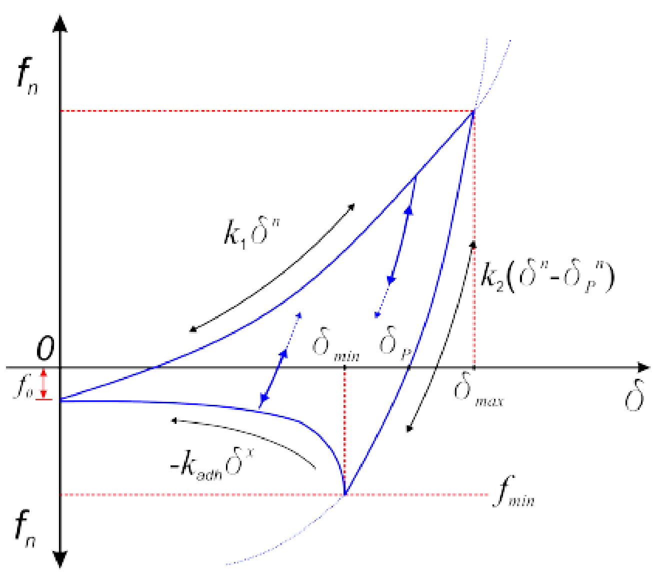

Figure 7. EEPA non-linear normal contact force-overlap relationship [26]

The normal contact force-overlap function for the EEPA contact model is illustrated in Figure 7. The model is defined by the following input parameters:

- **Constant pull off force, f_0 [N]**
  Used to model Van der Waals type force at the meso-scale

- **Surface energy, $\Delta\gamma$ [J/m^2]**
  A compound parameter, which models diverse stress-dependent micro-mechanical adhesive forces.

- **Contact plasticity ratio, λ_p**
  Level of plasticity used in the model.

- **Slope exponent, n**
  Defines the order of the normal force-overlap function. Both linear and non-linear functions are possible.

- **Tensile exponent, X**
  Defines the decay rate of the stress-dependent adhesive force.

- **Tangential stiffness multiplier, ζ_{tm}**
  Defines the frictional traction mobilization rate.

Morrissey, 2014 [26] presents a complete guide to the EEPA contact model including the formulation of normal and tangential contact forces.

5. NORMAL BEVAMETER SIMULATION PROCEDURE

To capture bevameter normal pressure-displacement response a robust, parameterizable and repeatable simulation procedure was developed. Simulations were run using the commercial DEM software, EDEM. Particle-particle contacts were modelled using the EEPA contact model due to the models ability to capture physical soft soil macro-mechanical behavior and for its computationally efficient meso-scopic modelling approach.

The simulation procedure is detailed in the following steps:

1. Generate DEM particles using a random static factory. Allow particles to settle inside an enclosed box geometry. Trim top layer particles to a parameterized top layer height.

2. Using a 'CompressionPlate' geometry, compress the bed of material to a defined pre-compression height.

3. Engage 'Bevameter' geometry with pre-compressed bed of material at a fixed velocity.

4. Export bevameter force and position and material bed porosity results for post processing.

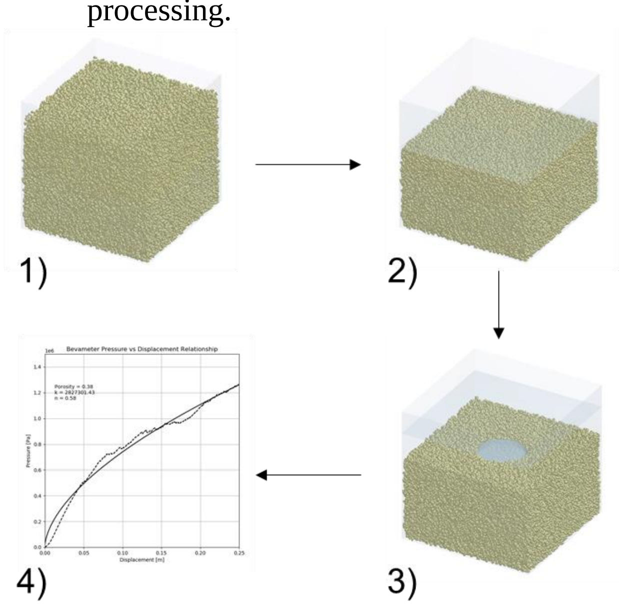

Figure 8. Bevameter simulation procedure

Top layer height, pre-compression height, bevameter velocity and overall simulation time are parameterized inputs controlled by a custom EDEM Coupling Interface application.

The enclosed box geometry was defined with edge lengths equal to 1.2 m and top layer height was defined equal to 0.8 m. Bevameter velocity was defined equal to 0.025 m/s. Simulations were run for a total of 11.5 s total simulation time including a 1.5 s setup stage (steps 1 and 2 detailed above) and 10 s where the bevameter was engaging with the bed of material. Simulation timestep of 3.5e-5 s and data save interval of 0.01 s were used for all simulations.

Constant particle shape, and size distribution was maintained for all simulations. Three equidistant 0.01 m radii spheres made up the multi-sphere particle. The triple sphere particle was selected because it limits rolling on all axes and encourages particle interlocking in consolidated beds of material. Variation in particle scale was introduced using a normal size distribution with mean equal to 1, standard deviation equal to 0.1, upper and lower caps equal to 1.25 and 0.75 respectively were used to constrain the size distribution.

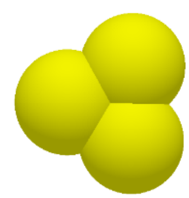

Figure 9. Triple sphere particle

The variable simulation input parameters were particle-particle static friction (interaction property input) and particle-particle constant pull off force (physics property input).

Input Parameter	**Value**
Particle density [kg.m^{-3}]	2500
Shear modulus [Pa]	5e6
Restitution	0.5
Static friction	0.2 to 0.8
Rolling friction	0.01
Constant pull off force [N]	0 to -60
Surface energy [J.m^{-2}]	0
Contact plasticity ratio	0.5
Slope exponent	1.5

Tensile exponent	1.5
Tangential stiffness multiplier	0.666667

Table 3. Simulation input parameter values

Step 2 of the normal bevameter simulation procedure, was used to nominally control soil particle packing. The pre-compression height was varied between 0.0 – 0.2 m in increments of 0.025 m.

As detailed in section 2.1, to solve for the Bekker-Wong normal parameters, k_c and k_φ, the pressure-displacement response for two different bevameter diameters are required and so the set of simulations detailed in this section were run with bevameter diameters 0.2 m and 0.4 m.

Sixteen combinations of material input parameters were run at nine different pre-compression heights for two bevemeter diameters - 288 simulations in total.

6. RESULTS

Bevameter force and position and material bed porosity were exported for all simulations. Dividing by bevameter area the pressure was calculated. Displacement is defined as zero at the first data save point where pressure is non-zero. Porosity values for the beds of material are reported at the first data save point where pressure is non-zero. It is convenient to use porosity (varying between 0 and 1), a measure of the volume of voids relative to the volume of solids in a bulk material, to describe particle arrangement for consolidated beds of material such as soft soils.

Simulation pressure-displacement results were used to determine the k and n best fit parameters for the Berinstein form of the p-z equation numerically using non-linear least squares approach to numerically minimize the error given by;

$$\underset{k,n}{argmin} \sum [p(z) - kz^n]^2 \qquad (5)$$

Pressure-displacement response for all soil models match closely the Berinstein form of the p-z equation. Porosity values varied from 0.50 to 0.24, values for k varied from ~ 400 kN/m^{n+2} to ~ 3400 kN/m^{n+2}, and values for n varied from 0.45 to 1.05. Two soil models across the parameter space displayed responses with $n > 1$. All other soil models displayed responses with $n < 1$ consistent with weakening, failing soils.

A sub-set of results, related to the sixteen material input parameter combinations are displayed in Figure 10 to 13 for pre-compaction heights of 0.100 m and 0.200 m and bevameter diameters of 0.2 m and 0.4 m. Constant pull off force 'POF' increases from left to right (0.0 N to -60.0 N) and static friction 'SF' increases from top to bottom (0.2 to 0.8). Similar response curves have been produced for all initial pre-compression height states.

Considering variable input parameters separately; both increasing pre-compression height and constant pull off force positively contributed to the stiffening of the pressure displacement response. Varying static friction has a less significant impact on the pressure-displacement response. Increasing pre-compression height reduces porosity as expected. Increasing static friction increases porosity. There is a step change in porosity when introducing constant pull off force which stabilizes for $f_0 < -20.0$ N.

Apart from cases where constant pull off force, $f_0 = 0.0$ N, when comparing results of the two bevameter diameters which simulations were run there is a trend showing the 0.2 m diameter bevameter produces a stiffer response vs the 0.4 m diameter bevameter. This is observed consistently across all material bed pre-compaction height states and for all material input parameter combinations with the abovementioned $f_0 = 0.0$N the exception. This is in contradiction to what is observed in the NG-NRMM CDT tests (Figure 3). It is proposed that this behavior is driven by an edge effect at the circumference of the bevameter where particle contact networks at the edge of the bevameter have a higher net contribution to the overall force exerted on the bevameter with decreasing bevameter diameter. The edge effect is more pronounced with increasing pull off force. The relatively large particles used in the simulation, approximately 10 to 1000 times the scale of real particles due to computational requirements, are likely a significant driving factor of this edge effect.

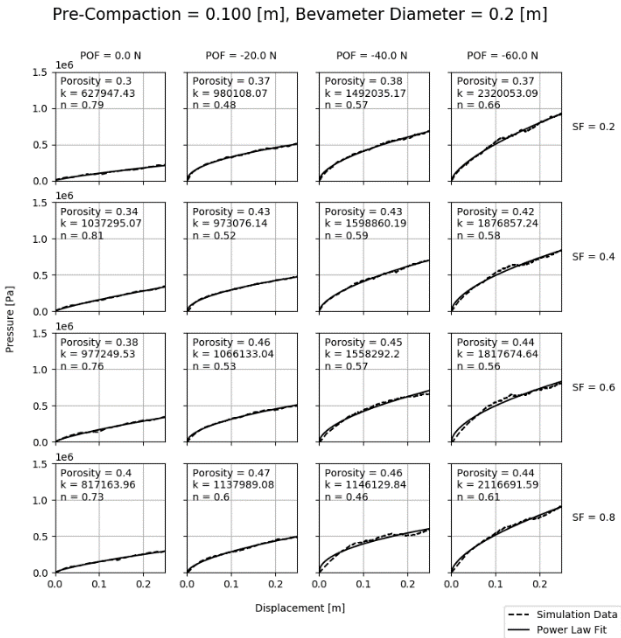

Figure 10. Pre-compression height = 0.100 m, bevameter diameter = 0.2 m

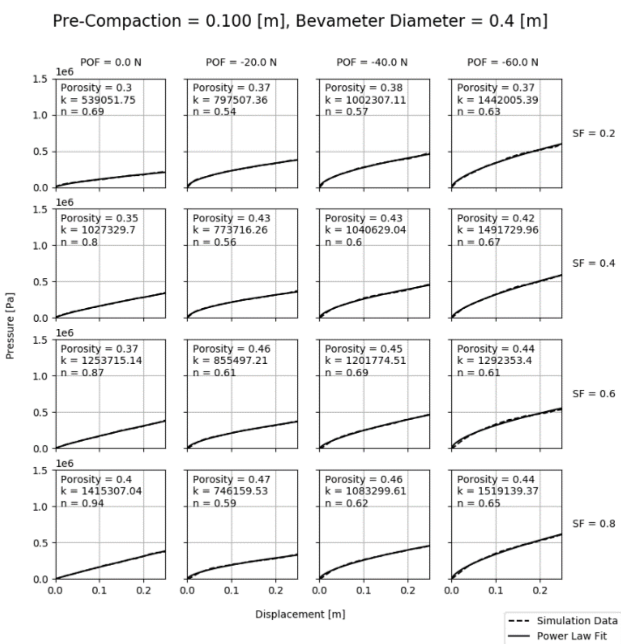

Figure 11. Pre-compression height = 0.100 m, bevameter diameter = 0.4 m

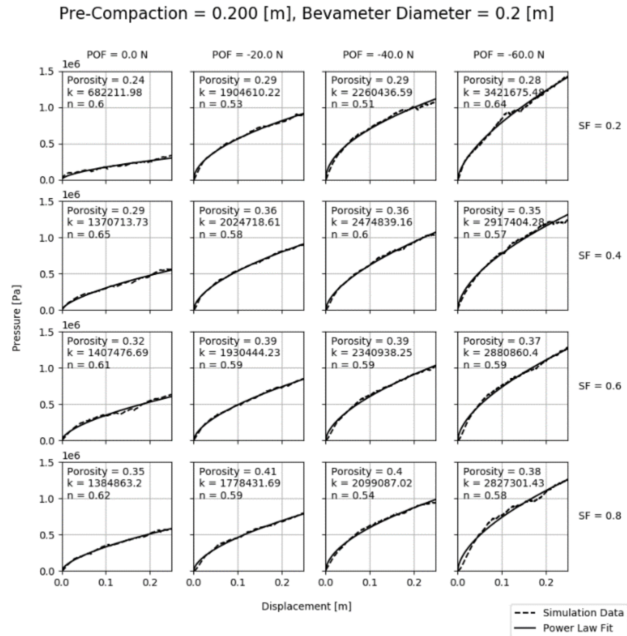

Figure 12. Pre-compression height = 0.200 m, bevameter diameter = 0.2 m

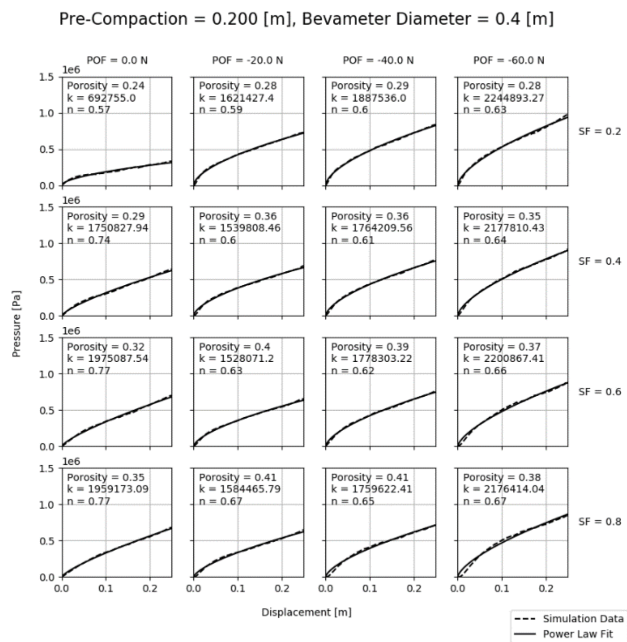

Figure 13. Pre-compression height = 0.100 m, bevameter diameter = 0.4 m

6.1. Relating field data to simulation results

It is convenient to be able to relate measured in-situ field data responses to simulated soil responses to enable a mechanism for selecting a DEM material model which provides a response consistent with that of the real-world soil.

The observed divergence in stiffness response between bevameter diameter sizes including varying n exponents makes it difficult to solve for the Bekker-Wong k_c and k_φ values for relating simulated soils directly to real world soils via these parameters. It is seen that simulated soil responses follow closely the original Berinstein form of the p-z equation and with few exceptions produce results consistent with n < 1 soil responses. It is proposed that, for real world soil responses with n < 1 and 400 kN/m^{n+2} < k < 3400 kN/m^{n+2}, it is possible to search the database of simulated soil responses and find an adequate match for the real world soil response. It is then possible to use this material model in DEM + MBD simulations to assess, qualitatively and quantitatively, vehicle performance in the simulated soft soil environment.

The suggested process of selecting an accurate DEM material model is outlined in the following steps

1. For a given real world soft soil response solve for k_r and n_r (Equation 5).

2. Select the DEM material model by computing the minimum absolute difference for the integral of the real-world vs simulated p-z curves (Equation 6).

$$\underset{i}{argmin}\left|\int k_r z^{n_r} dz - \int k_i z^{n_i} dz\right| \quad (6)$$

Following this process, Figure 14 shows the best fit simulated soil response against a real world response selected from the set of Variable Hill Climb, Dry material responses in the NG-NRMM CDT data. Simulated soil responses for bevameter diameter 0.4m were used to compute the simulated p-z integral. The real-world soil response k and n values were calculated as 1986 kN/m^{n+2} and 0.69 respectively. The simulated soil k and n values were calculated as 1884 kN/m^{n+2} and 0.67 respectively. Pre-compression height, porosity and simulation input parameters for the identified response are displayed in Table 4.

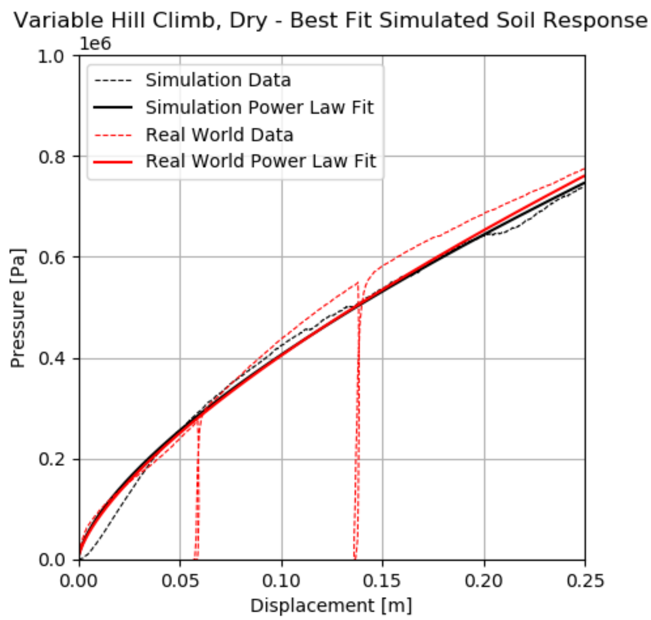

Figure 14. Variable hill climb, dry - best fit simulated soil response

Parameter	Value
Pre-compaction height [m]	0.150
Porosity	0.40
Particle density [kg.m^{-3}]	2500
Shear modulus [Pa]	5e6
Restitution	0.5
Static friction	0.6
Rolling friction	0.01
Constant pull off force [N]	-60
Surface energy [J.m^{-2}]	0
Contact plasticity ratio	0.5
Slope exponent	1.5
Tensile exponent	1.5
Tangential stiffness multiplier	0.666667

Table 4. Variable hill climb, dry - best fit simulated soil simulation parameters

7. CONCLUSIONS

A simulation procedure has been developed for reproducing the uniaxial bevameter compression test widely applied for characterizing the normal pressure-displacement response for soft soils. Numerical methods have been applied to extract the k and n best fit parameters for the Bernstein form of the p-z equation. Results have been catalogued reporting the porosity, k and n values. Plots of the raw and power law best fit curves have been included. Soil shear response has not been modelled as part of this study and should be considered in future work. Methods for relating in-situ field data to the catalogue of simulated material bed responses have been proposed and an example referencing the Variable Hill Climb, Dry soil response reported in the NG-NRMM CDT has been demonstrated. Single tire and/or full vehicle DEM + MBD simulations should be performed and results compared against real world off-road mobility events to further validate the DEM soft soil material models presented in this work.

With few exceptions, the simulated material beds produced $n < 1$ shaped pressure-displacement responses typical of weakening, failing soils. The responses produced fit the idealized Bernstein form of the p-z equation exceptionally well. It is reasonable to infer the homogeneity of the simulated beds of material influences this behavior. Further research is required to understand how best to capture $n > 1$ shaped responses with multi-layered beds of material, being a potential area to concentrate effort.

There is disparity in results between the two bevameter sizes simulated and it has been proposed that particle size and bevameter edge effect is influencing this. Considering this with the evidence from the experimental research community which shows bevameter size and shape to have a significant influence on the normal pressure-displacement response it is challenging to draw direct comparisons between field data and these simulation results. This is further exacerbated by ambiguity involved in curve fitting to noisy experimental data which often does not follow the idealized Bekker-Wong forms of the p-z equations. Values for the k_c and k_φ Bekker-Wong parameters have not been produced, largely because of these challenges.

Particle shape, size, density, shear modulus and other simulation input parameters have not been varied as part of this study. In the future the parameter space could be expanded to consider more of these input parameters. Other methods for extending the parameter space such as Reduced Order Models or Machine Learning could also be considered as alternatives to running full factorial parameter space analysis.

8. References

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