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NEW FINITE ELEMENT / MULTIBODY SYSTEM ALGORITHM FOR MODELING FLEXIBLE TRACKED VEHICLES

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ABSTRACT

The dynamic simulation of multibody tracked vehicles offers engineers a powerful tool with which they may analyze and design. Currently, parts of these complex mechanisms are introduced to multibody algorithms as rigid bodies. Then in a follow-on structural analysis, the loads from the multibody dynamic simulation are input to calculate strains and stresses within the bodies. The present investigation aims to establish appropriate means by which flexible three-dimensional track links, which allow large relative rotation between the elements, can be modeled. This will pave the way towards the incorporation of detailed flexible structural models into a multibody dynamic simulation environment allowing for an integrated solution. In addition, a new formulation for the interaction between the rigid sprocket teeth and flexible chain is presented. Numerical results are introduced to illustrate the effects of flexible links on the dynamics of tracked vehicles.

INTRODUCTION

Current commercial implementations for tracked vehicle modeling are not ideal for the analysis of three-dimensional flexible chain tracked vehicles. Each track chain may contain large number of separate bodies/elements which require connectivity conditions be imposed at each time step.

Non-multibody simulation models have had success in providing engineers with a means with which to quantitatively compare tracked vehicles [1-3]. It is hoped, however, that multibody simulation models will provide the working engineer tracked vehicle models based on fewer assumptions and greater flexibility of simulation.

It is one of the objectives of this investigation to present the results of incorporating the recently developed Absolute Nodal Coordinate Formulation (ANCF) C^0/C^1 flexible chain into a tracked vehicle multibody model. The C^0/C^1 chain model offers a number of advantages over current chain models. First, the linear connectivity conditions resulting from ANCF may be applied at a preprocessing stage [5]. This allows for an efficient elimination of the dependent variables. When used in conjunction with ANCF spatial finite elements, the linear connectivity conditions lead to a constant inertia matrix, as well as zero Coriolis and centrifugal forces. Further, instead of considering a series of interconnected rigid or flexible bodies/elements, the new formulation allows for the use of one finite element mesh for the chains despite the finite rotations of the track links with respect to each other and relative translation between the two track chains. Numerical examples will be used to illustrate the advantages of the C^0/C^1 flexible chain formulation as relevant to tracked vehicle models.

Another objective of this investigation is to present a newly developed formulation for the contact between the C^0/C^1 chain model and the sprocket teeth. This interaction formulation will allow the building of tracked vehicle models that can more accurately represent the complex dynamics of tracked vehicles.

The organization of the paper is as follows. The first section contains a summary of the absolute nodal coordinate formulation (ANCF), as well as a presentation of the beam element used in this investigation. In the next section, the constrained multibody dynamic equations of motion are presented. The third section summarizes the recently developed finite element ANCF mesh which can be used to model flexible chains. The fourth section presents the models used for the interaction between the different parts of the tracked vehicle running gear. Specifically, the fourth section will introduce a new formulation for the simulation of sprocket-flexible chain interaction.

1. ABSOLUTE NODAL COORDINATE FORMULATION

In this investigation, the finite element absolute nodal coordinate formulation is used in the modeling of flexible bodies. The global position of an arbitrary point on element *e* of body *i* can be defined (figure 1) in an inertial global coordinate system *XYZ* as

$$\mathbf{r}^{ie} = \mathbf{S}^{ie}(x^{ie}, y^{ie}, z^{ie})\mathbf{e}^{ie} \quad (1)$$

where $\mathbf{S}^{ie}(x^{ie}, y^{ie}, z^{ie})$ is the space-dependent matrix that defines the element shape functions, and $\mathbf{e}^{ie}$ is the time-dependent vector of nodal coordinates.

The vector of nodal coordinates $\mathbf{e}^{ie}$ consists of nodal displacements and gradients. It can be defined as

$$\mathbf{e}^{ie} = \left[\mathbf{r}^{ie1\mathrm{T}} \ \frac{\partial \mathbf{r}^{ie1\mathrm{T}}}{\partial x^{ie}} \ \frac{\partial \mathbf{r}^{ie1\mathrm{T}}}{\partial y^{ie}} \ \frac{\partial \mathbf{r}^{ie1\mathrm{T}}}{\partial z^{ie}} \ \mathbf{r}^{ie2\mathrm{T}} \ \frac{\partial \mathbf{r}^{ie2\mathrm{T}}}{\partial x^{ie}} \ \frac{\partial \mathbf{r}^{ie2\mathrm{T}}}{\partial y^{ie}} \ \frac{\partial \mathbf{r}^{ie2\mathrm{T}}}{\partial z^{ie}}\right]^{\mathrm{T}} \quad (2)$$

In equation (2), $\mathbf{r}^{iek\,\mathrm{T}}, \frac{\partial \mathbf{r}^{iek\,\mathrm{T}}}{\partial x^{ie}}, \frac{\partial \mathbf{r}^{iek\,\mathrm{T}}}{\partial y^{ie}}$, and $\frac{\partial \mathbf{r}^{iek\,\mathrm{T}}}{\partial z^{ie}}$ refer to the displacements and gradients at node *k*. It can be shown that the selection of nodal positions and gradients as element coordinates leads to an exact modeling of rigid body dynamics. The shape function for the three-dimensional beam element based on the absolute nodal coordinate formulation used in this investigation is defined by

$$\mathbf{s}^{ie} = [s_1^{ie}\mathbf{I} \quad s_2^{ie}\mathbf{I} \quad s_3^{ie}\mathbf{I} \quad s_4^{ie}\mathbf{I} \quad s_5^{ie}\mathbf{I} \quad s_6^{ie}\mathbf{I} \quad s_7^{ie}\mathbf{I} \quad s_8^{ie}\mathbf{I}] \quad (3)$$

where **I** is the 3x3 identity matrix and the shape functions s_k^{ie}, $k = 1,...,8$ are given as follows [4]:

$$\begin{aligned} &S_1 = 1 - 3\xi^2 + 2\xi^3,\ S_2 = l(\xi - 2\xi^2 + \xi^3),\ S_3 = l\eta(1-\xi) \\ &S_4 = l\varsigma(1-\xi),\ S_5 = 3\xi^3 - 2\xi^3,\ S_6 = l(-\xi^2 + \xi^3) \\ &S_7 = l\eta\xi,\ S_8 = l\varsigma\xi \end{aligned} \quad (4)$$

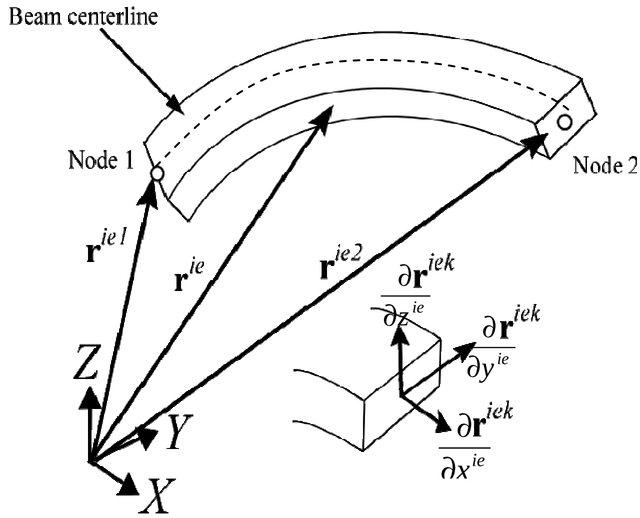

Figure 1: ANCF Beam Element Coordinates

In equation (4), $\xi = x^{ie}/l$, $\eta = y^{ie}/l$, $\varsigma = z^{ie}/l$, and l is the length of the element. It can be shown that the three dimensional beam element presented here is capable of capturing the effects of shear deformation and rotary inertia as well as the coupling between the cross section deformation and bending/extension.

2. CONSTRAINED EQUATIONS OF MOTION

Using the principle of virtual work, one may write the finite element equations of motion in terms of the vector of nodal coordinates as follows:

$$\mathbf{M}^{ie}\ddot{\mathbf{e}}^{ie} = \mathbf{Q}_k^{ie} + \mathbf{Q}_e^{ie} \quad (5)$$

In equation (5), $\mathbf{Q}_k^{ie}$ is the vector of generalized element elastic forces, $\mathbf{Q}_e^{ie}$ is the vector of generalized element external forces, and $\mathbf{M}^{ie}$ is the constant mass matrix of the element.

Using equation (5), one may write the equations of motion of the constrained multibody system in the following form:

$$\left.\begin{aligned} \mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}_{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\lambda} = \mathbf{Q} \\ \mathbf{C}(\mathbf{q}, t) = 0 \end{aligned}\right\} \quad (6)$$

where **M** is the system mass matrix, **q** is the vector of the system coordinates (which includes the absolute nodal coordinates and other coordinates used to describe the motion of the rigid bodies), **λ** is the vector of Lagrange multipliers, **Q** is force vector, **C** is the vector of kinematic constraint equations, and $\mathbf{C_q}$ is the Jacobian matrix of constraint equations.

3. NEW ANCF FINITE ELEMENT MESH

Fully parameterized absolute nodal coordinate formulation finite elements have 12 degrees of freedom. These degrees of freedom include: three rigid body translations, three rotations, and six deformation modes. We may introduce a new formulation for a pin joint that uses the following six scalar equations defined at the joint node between bodies *i* and *j* [5]:

$$\mathbf{r}^i = \mathbf{r}^j \tag{7}$$

$$\frac{\partial \mathbf{r}^i}{\partial \alpha} = \frac{\partial \mathbf{r}^j}{\partial \alpha} \tag{8}$$

In the above equations, α is the coordinate line that defines the joint axis. It can be either *x, y, z,* or any other coordinate line since the use of ANCF makes easy the use of the gradient tensor transformation. Consider, for example, the gradient transformation between *u, v, w* and *x, y, z*. This transformation is given by $\begin{bmatrix}\mathbf{r}_u & \mathbf{r}_v & \mathbf{r}_w\end{bmatrix} = \begin{bmatrix}\mathbf{r}_x & \mathbf{r}_y & \mathbf{r}_z\end{bmatrix}\mathbf{A}$ where **A** is given by the constant matrix shown below.

$$\mathbf{A} = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{bmatrix} \tag{9}$$

The six scalar equations represented by equations (7) and (8) reduce the number of degrees of freedom for the new pin joint to six; one rotation, and five deformation mode degrees of freedom. As a result, we have C^1 continuity with respect to the α coordinate and C^0 continuity with respect to the remaining coordinates. The linear scalar equations (7) and (8) can be imposed at a preprocessing stage, thus allowing for the elimination of the dependent variables. These equations can be used to develop a new kinematically linear finite element mesh. This would allow arbitrarily large relative rotations between its constituent elements at the mesh joints.

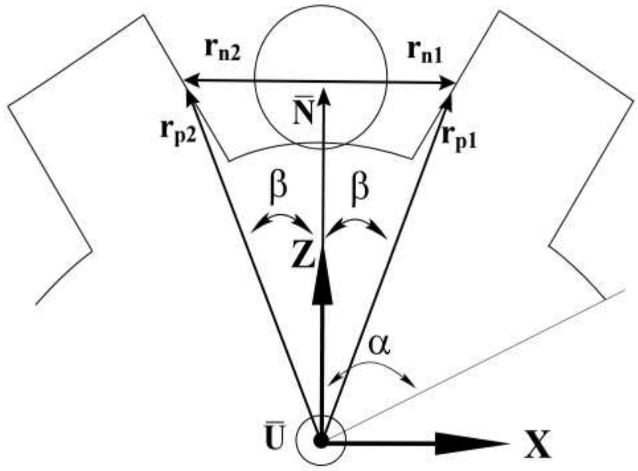

Figure 2: Geometry of contact between rigid sprocket teeth and the C^0/C^1 chain

4. FLEXIBLE CHAIN-RIGID COMPONENT CONTACT FORMULATIONS

The new finite element ANCF C^0/C^1 mesh formulation that was presented in the previous section can be used to more accurately describe, with one mesh, a flexible track. In this section the contact between the flexible track links, sprockets, rollers, and idlers will be described. The idlers, sprockets, and rollers will be modeled as rigid bodies in this investigation.

While previous work has focused on modeling flexible belt driven vehicles [6], the present investigation differs in that it includes a formulation for the interaction between the flexible chain and sprocket teeth. This will allow for future investigations in which the dynamics of rigid and flexible link tracked vehicles can appropriately be compared.

The formulation for the interaction between the sprocket teeth and the C^0/C^1 chain is as follows (one can refer to figure 2 for the definitions of the variables used): let $\bar{\mathbf{N}}$ be the vector that defines the local position of the center of the pin joint in the sprocket coordinate system, $\bar{\mathbf{U}}$ is a vector which defines the axis around which the sprocket rotates, r_{pin} is the pin radius, l_B is the backlash as a percentage of the pin radius r_{pin}, n_t is the current tooth number, and T_n is the number of teeth. Note that it is assumed that the sprocket axis is parallel to the Y-axis. Take angle α as from the center line of the tooth to the vertical center line of the sprocket. The two angles α and β can be calculated from geometry as

$$\alpha = \left(n_t - 1\right)\frac{2\pi}{T_n} \tag{10}$$

$$\beta = \tan^{-1}\left(\frac{\left(1+l_B\right)r_{pin}}{\left|\bar{\mathbf{N}}\right|}\right) \tag{11}$$

One can then find the vectors that define the location of the contact points between the sprocket teeth and the pin as

$$\mathbf{r}_{p1} = \left|\mathbf{r}_{p1}\right|\left[\cos\left(\alpha+\beta\right) \quad 0 \quad \sin\left(\alpha+\beta\right)\right]^T \tag{12}$$

$$\mathbf{r}_{p2} = \left|\mathbf{r}_{p2}\right|\left[\cos\left(\alpha-\beta\right) \quad 0 \quad \sin\left(\alpha-\beta\right)\right]^T \tag{13}$$

where the magnitude of $\mathbf{r}_{p1}$ and $\mathbf{r}_{p2}$ can be calculated from equation (14).

$$\left|\mathbf{r}_{p1}\right| = \left|\mathbf{r}_{p2}\right| = \sqrt{\left(\bar{\mathbf{N}}^2 + \left(1+l_B\right)^2\left(r_{pin}\right)^2\right)} \tag{14}$$

The distance between the pin and the tooth contact point is given by equations (15-16).

$$\mathbf{r}_{n1} = \mathbf{r}_{p1} - \bar{\mathbf{N}} \tag{15}$$

$$\mathbf{r}_{n2} = \mathbf{r}_{p2} - \bar{\mathbf{N}} \tag{16}$$

The condition for contact is given by

$$\left|\mathbf{r}_{n1}\right| \le r_{\text{pin}} \quad \text{or} \quad \left|\mathbf{r}_{n2}\right| \le r_{\text{pin}} \tag{17}$$

If the conditions for contact are true, the force normal to the contact surface is evaluated using a continuous force model defined as follows

$$\mathbf{F}_n = (kd + c\dot{d})\hat{\mathbf{V}} \tag{18}$$

where d can be found from the following equation

$$d = r_{pin} - \left|\mathbf{r}_{n1}\right| \quad \text{or } d = \left|\mathbf{r}_{n2}\right| - r_{pin} \tag{19}$$

and the vector normal to the contact surface is easily found from

$$\bar{\mathbf{V}} = \bar{\mathbf{U}} \times \bar{\mathbf{N}} \tag{20}$$

With the tooth contact force established, one also needs to include the contact force due to the sprocket base circle and pin contact. Figure 3 illustrates the geometry used to define the contact between the C^0/C^1 chain and the cylindrical rigid body that constitutes the sprocket base circle [6].

The radial (normal) penetration for the configuration shown in figure 3 can be formulated as follows:

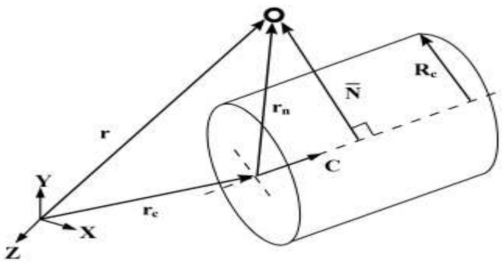

Figure 3: Geometry of contact between rigid cylinder and the C^0/C^1 chain

$$d = R_c - \left|\bar{\mathbf{N}}\right| \tag{21}$$

where R_c is the radius of the cylinder and $\bar{\mathbf{N}}$ is a vector that defines the position of the contact point in the coordinate system of the cylinder. $\bar{\mathbf{N}}$ can be defined as

$$\bar{\mathbf{N}} = \mathbf{r}_n - \left(\mathbf{r}_n^T \mathbf{c}\right)\mathbf{c} \tag{22}$$

The normal force for the contact between the rigid cylinder and the C^0/C^1 chain can be then be found by using the appropriate penetration and rate of penetration in equation (18).

The tangential contact force is based on a trilinear Coulomb friction law characterized by the following equations

$$\mathbf{F}_{\text{t}} = -\mu\left|\mathbf{F}_n\right|\mathbf{t}, \qquad \left|\mathbf{V}_{\text{t}}\right| > v_0 \tag{23}$$

$$\mathbf{F}_{\text{t}} = -v_s\left|\mathbf{V}_{\text{t}}\right|\mathbf{t}, \qquad \left|\mathbf{V}_{\text{t}}\right| \le v_0 \tag{24}$$

where $\mathbf{V}_t$ is the tangential velocity, v_s is a friction parameter that defines the slope in the transition area, and the constant v_0 is defined as

$$v_0 = \frac{\mu\left|\mathbf{F}_n\right|}{v_s} \tag{25}$$

The total force will then be the summation of the normal and tangential forces. In the case of the idlers and rollers, the contact forces will be calculated in a manner similar to the contact between the sprocket base circle and the C^0/C^1 chain.

5. NUMERICAL RESULTS

The previous section presented the formulations used in modeling the contact between the rigid sprockets, rollers, idlers, and the C^0/C^1 track or chain. In this section, numerical results of a model of a rubber tracked vehicle that includes only one track are presented. The model contains a ground, chassis, sprocket, idler, two rollers, and a flexible track. The mass of the driving sprocket and the idler are assumed to be 40 kg. The inertia of driving sprocket and the idler about the X, Y, and Z axes are respectively 0.4 kg.m^2, 0.8 kg.m^2 and 0.4 kg.m^2. The mass of the rollers is 10 kg. The mass of the vehicle chassis is 100 kg. In the numerical example presented in this section, the chassis is assumed to have zero initial velocity. The angular velocity constraint that is applied to the sprocket is defined by the following equation:

$$\omega = \begin{cases} 0 & t <= 0.2 \\ 100\dfrac{(t-0.2)}{0.8} & 0.2 < t <= 1 \\ 100 & t > 0.2 \end{cases} \qquad (26)$$

where ω is the rotational speed of sprocket in rad/s and t is the time in seconds.

The diameter of the base circle of the sprocket is 0.38 m and has 10 teeth. The diameter of the idler and the rollers are respectively 0.38 m and 0.18 m. The chain is modeled using 24 ANCF elements that have a rectangular cross section of dimensions $0.02\text{m}\times 0.4\text{m}$ and density of 2000 kg/m^3. The incompressible Neo-Hookean model and non-linear damping model are used to model the internal behavior of the rubber chain. The Neo-Hookean constant is assumed to be 1×10^8, the incompressibility constant is assumed to be 1×10^9, and the dilatation and deviatoric dissipation factors used for the damping model are assumed to be 1×10^{-4} and 5×10^{-5}, respectively. A pretension positive axial strain of value 0.05 is applied to the rubber chain. A compliant force model is used to describe the contact between rigid bodies and the chain. The stiffness and damping coefficients used in the rigid bodies and chain contact force model are given, respectively, by 9×10^7 N/m^3 and 2×10^3 N.s/m^3. Tangential friction forces are also introduced using a coefficient of dry friction of 1.2. The friction parameter that defines the slope in the transition region is assumed to be 10^6 N.s/m^3. Figure 4 shows the X, Y and Z coordinates of the position vector of the center of mass of the chassis. Figure 5 shows the velocity of the chassis in the X-direction. Note that in figure 6 the displacement decreases at certain times; this is an indication of sliding behavior, which will continue until the rubber elements in contact with the ground obtain zero speed in the x direction.

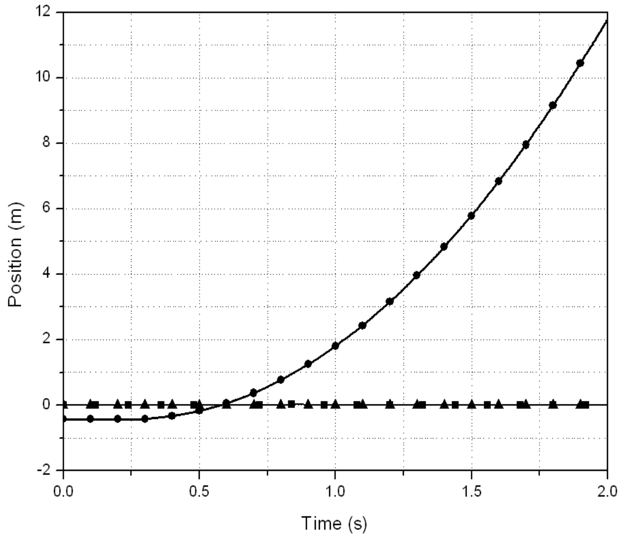

Figure 4: Position of Center of Mass of the Chassis (—●— X coordinate; —■—Y coordinate; —▲— Z coordinate)

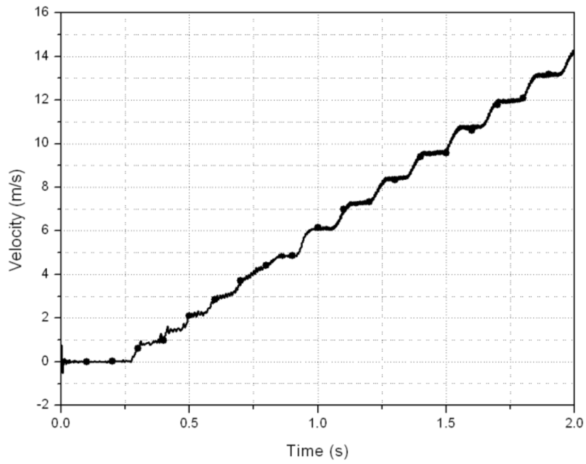

Figure 5: X velocity of the center of mass of the chassis

Figures 6 and 7 show, respectively, the X and Z coordinates of a point on the rubber chain that is initially located at the top of the idler, as function of time. Figure 8 shows the motion trajectory of this point relative to the moving center of mass of the chassis.

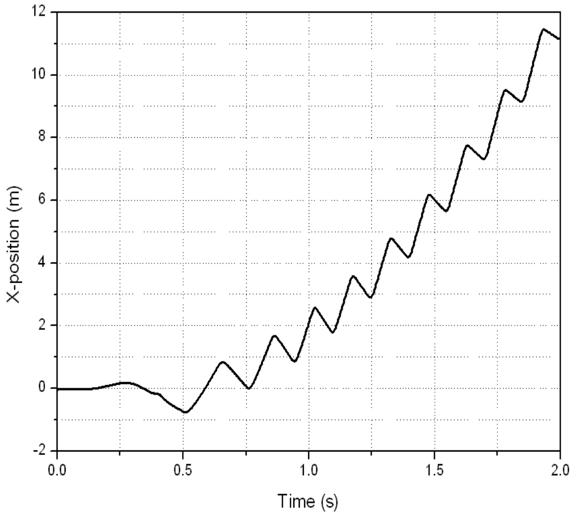

Figure 6: X coordinate of the position vector of a point on the rubber chain

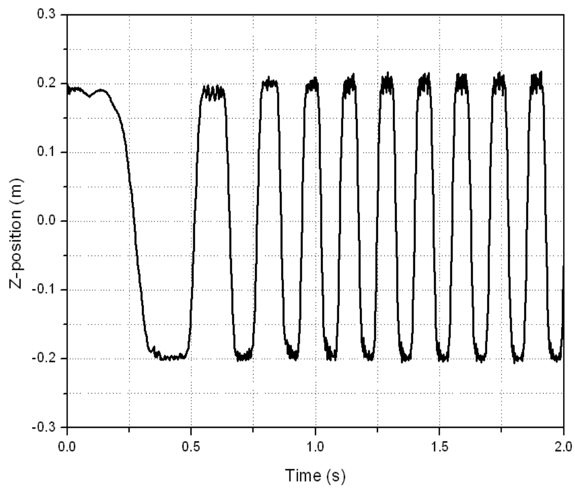

Figure 7: Z coordinate of the position vector of a point on the rubber chain

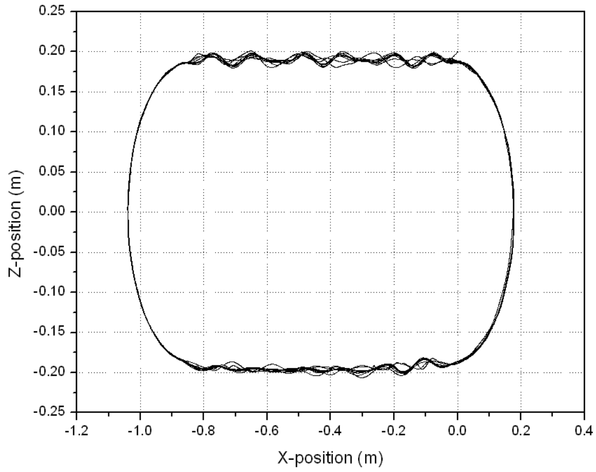

Figure 8: Trajectory of the position vector of a point on the rubber chain

6. SUMMARY AND CONCLUSIONS

The present investigation proposed a method to model the contact between spatial track chain links and rigid running gear components. The Absolute nodal coordinate formulation was used to model the C^0/C^1 track or chain using linear connectivity conditions. This chain formulation allows for C^1 continuity in the direction of the pin joint definition and C^0 continuity with respect to the other components. A new formulation for the compliant force contact between three dimensional flexible track links and rigid sprocket was presented. Along with the formulation for the contact between the rigid rollers, idlers, and flexible links, these contact formulations were used to model a tracked vehicle with a single track. This model may then be used in future investigations to compare the dynamics of rigid and flexible tracks in tracked vehicle models.

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