**2014 NDIA GROUND VEHICLE SYSTEMS ENGINEERING AND TECHNOLOGY SYMPOSIUM**
**MODELING & SIMULATION, TESTING AND VALIDATION (MSTV) TECHNICAL SESSION**
**AUGUST 12-14, 2014 - NOVI, MICHIGAN**

# IOWA DEVELOPED RELIABILITY-BASED DESIGN OPTIMIZATION (I-RBDO) - TECHNOLOGY TRANSFER

**K.K. Choi, Nicholas J. Gaul, Hyeongjin Song, and Hyunkyoo Cho**
Department of Mechanical and Industrial Engineering
The University of Iowa
Iowa City, IA

**David Lamb and David Gorsich**
U.S. Army Tank Automotive Research, Development & Engineering Center (TARDEC)
Warren, MI

## ABSTRACT

*The recent U.S. Army TARDEC's 30-Year Strategy calls for enhancing their skill set in the "ilities," especially reliability, since this factor directly impacts more than 58% of life cycle costs, according to a DoD study. To support this initiative, this paper presents technology transfer of Iowa developed Reliability-Based Design Optimization (I-RBDO) software by integrating theories and numerical methods that have been developed over a number of years in collaboration with the Automotive Research Center (ARC), which is funded by the U.S. Army TARDEC. Both the sensitivity-based and sampling-based methods for reliability analysis and design optimization methods are integrated in I-RBDO for broader multidisciplinary applications. I-RBDO has very comprehensive capabilities that include modeling of input distributions for both independent and correlated variables; a variable screening method for high dimensional RBDO problems; statistical analysis; reliability analysis; RBDO; and confidence-based RBDO. The research software I-RBDO has been provided to selected academia, industry, TARDEC HPC and Army Research Lab (ARL) Dedicated Support Partition (DSP) for testing its effectiveness by applying to various test problems. The Iowa team will continue working with TARDEC and ARL using I-RBDO on HPC and DSP, respectively, for Army applications of I-RBDO. With successful applications of I-RBDO, the Iowa team established a small start-up company to develop a commercial Reliability Analysis & Multidisciplinary Design Optimization (RAMDO) software. The company was awarded an Army SBIR funding contract in May 2014. This commercialization of the ARC funded I-RBDO software represents a major technology transfer for the benefit of the Army. The success of TARDEC led research and development is highlighted by taking a significant multi-year project like this and demonstrating a full transition of the technology to the commercial marketplace.*

## 1. INTRODUCTION

It is critical to the US Army to have reliable ground vehicles with optimized weights that can be relied on to demonstrate consistently high levels of performance for survivability, mobility, and durability under a wide range of operational conditions without being subject to unanticipated premature failure, and with substantially reduced maintenance requirements. In commercial manufacturing industries for passenger vehicle, light commercial vehicle, heavy truck, bus, aircraft, space vehicle, heavy construction equipment, farm equipment, ship, wind energy, etc., developing and producing optimized reliable products are the primary goals for success of their businesses and to reduce warranty costs. As the computer-aided design (CAD) and computer-aided engineering (CAE) tools are advancing, the simulation-based design process is often used to obtain an optimum design, prior to prototype development, to reduce the product development cost. However, a design that is deterministically optimized without inclusion of reliability will be most likely around 50% reliable as shown in Figure 1.

For simulation-based design process, there is a need for capabilities of Process Integration and Design Optimization (PIDO) to support multidisciplinary analysis and reliability-based design optimization (RBDO). For PIDO, two key capabilities need to be developed: (1) Process Integration (PI) for seamless integration of diverse CAD/CAE tools for multidisciplinary analyses; and (2) Design Optimization (DO) for multidisciplinary RBDO. The first capability is focused on integration of CAD/CAE software tools, whereas the second capability is focused on computational methods to carry out multidisciplinary RBDO as shown in Figure 2. There are a number of commercial softwares that have very effective Process Integration (PI) capabilities for multidisciplinary simulation. However, computational capabilities for multidisciplinary RBDO are not as advanced yet in terms of accuracy and efficiency.

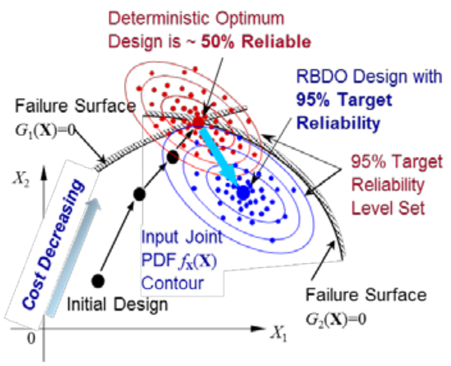



Figure 1. Deterministic Design Optimization (DDO) vs. RBDO

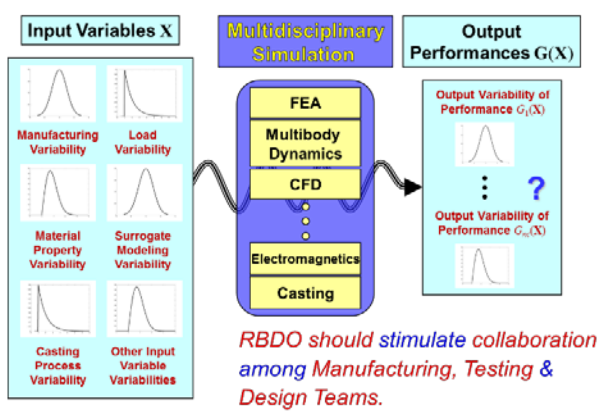



Figure 2. Multidisciplinary Simulation with Input Variability

Basic RBDO theories and numerical methods have been developed at the University of Iowa over a number of years in collaboration with the U.S. Army TARDEC. For the input distribution model, a weight-based Bayesian method with two-step procedure is developed using seven marginal probability density functions (PDFs) and nine joint copulas with correlation parameters by best fitting the data [1-4]. For variable screening, the variables that induce larger output variances are selected as important variables [5,6]. For the sensitivity-based RBDO method, the enriched performance measure approach (PMA+) is developed using the inverse first-order reliability method (FORM) [7,8]; whereas a higher-order reliability method is developed using the dimension reduction method (DRM) [9-13] for highly nonlinear constraints. For the sampling-based reliability analysis and design optimization, surrogate models are used [14-16]. For accurate surrogate models, newly enhanced accurate dynamic Kriging method (DKG) and efficient sequential sampling strategy are used [16]. The DKG method optimally selects the best surrogate model out of 21 candidate surrogate models. In addition to DKG, for efficiency of surrogate models, a hyper-spherical local window concept is developed for both independent and correlated input variables using the copulas and Rosenblatt transformation. The score functions derived from the marginal PDFs and copulas are used to obtain accurate stochastic design sensitivity [17,18]. Use of the score function provides an interesting option to support user generated surrogate models for reliability analysis and RBDO. In addition to the DKG method, for the sampling-based RBDO, the virtual support vector machine (V-SVM), which is a classification method, is developed [19]. For fast turn-around of the RBDO process, the proposed sampling-based RBDO method is mapped to a multiple core environment in High Performance Computing (HPC).

These basic theories and numerical methods are implemented and integrated to develop the Iowa developed Reliability-Based Design Optimization (I-RBDO) software with a user interface (UI) for technology transfer to industry, academia, and Army. The research software I-RBDO has been provided to selected academia, industry, U.S. Army TARDEC HPC and Army Research Lab (ARL) Dedicated Support Partition (DSP) for testing of its effectiveness by applying to various test problems. The Iowa team will continue working with TARDEC and ARL using I-RBDO on HPC and DSP, respectively, for Army applications of I-RBDO. I-RBDO has been successfully applied to optimize designs of a passenger vehicle for noise, vibration, harshness (NVH) and safety [20]; durability [21,22]; casting process design [23]; ship hydrodynamics; fluid-structure interaction [24]; welding design [25]; superconducting magnetic energy storage system [26]; electro-thermal polysilicon actuator [27]; etc.

The commercialization of a research software is one of the most effective means of technology transfer. With the success of I-RBDO by various users, the Iowa team established a small start-up company, RAMDO Solutions, LLC, in fall 2013. Using I-RBDO, the company will develop a commercial software Reliability Analysis & Multidisciplinary Design Optimization (RAMDO) and continue to support academia, industry, and Army. The company was awarded an Army SBIR funding contract in May 2014. This commercialization of the ARC funded I-RBDO software represents a major technology transfer for the benefit of the Army. The success of TARDEC led research and development is highlighted by taking a significant multi-year project like this and demonstrating a full transition of the technology to the commercial marketplace.

Since detailed description of all RBDO methods that have been developed by the Iowa team will make this paper too long, in the following sections some selected key RBDO methods will be briefly described along with applications of I-RBDO by the partners. Also, the commercialization status will be briefly explained.

## 2. MODELING DISTRIBUTIONS FOR INPUT VARIABLES

For reliability analysis and RBDO, the first capability that is needed is modeling the uncertainty for both input design variables and parameters such as material properties and/or loadings. For certain design variables or parameters, the input distributions may be well known. However, for many design variables or parameters, only limited experimental or test data may be available. Identification of the input uncertainty model with a limited data is challenging when input variables or parameters are correlated. Methods for modeling input distributions for both independent and correlated variables as well as parameters are developed in Refs. 1-4 using a Bayesian method, which selects a marginal cumulative distribution function (CDF) and/or copula with the highest normalized weight among candidates. Nine candidate copulas: Clayton, Ali-Mikhail-Haq (AMH), Gumbel, Frank, A12, A14, Farlie-Gumbel-Morgenstern (FGM), Gaussian and Independent, are used to model joint distributions. To measure the correlation between two random variables, Kendall's tau [3,4] is used, since unlike Pearson's rho, Kendall's tau does not assume that the relationship between two random variables is linear. For marginal distributions, seven candidate CDFs, Gaussian, Weibull, Gamma, Lognormal, Gumbel, Extreme, and Extreme Type-II, are used. If two variables are correlated, there are 7×7×9 = 441 possible combinations for the input distribution models, making it difficult to clearly identify the best model that fits the data. Thus, an efficient and accurate weight-based Bayesian method using a two-step procedure is developed for modeling input distributions [4], which performs significantly better than the Markov chain Monte Carlo (MCMC)-based Bayesian method.

As an example, Figure 3 shows result of the input distribution modeling of the highly correlated fatigue material property coupon test data of alloy steel SAE 950X using the weight-based Bayesian method and the two-step procedure [4]. The marginal PDFs of the fatigue strength coefficient $\sigma_f'$ and exponent $b$ follow lognormal and normal distributions, respectively; and the fatigue ductility coefficient $\varepsilon_f'$ and exponent $c$ follow lognormal and normal distributions, respectively. As shown in Figure 3, the fatigue strength coefficient and exponent are negatively correlated and the Gaussian copula best fits the data; whereas fatigue ductility coefficient and exponent are negatively correlated and the Frank copula best fits the data. As will be shown in Section 3, properly modeling PDFs of these correlated data is critically important to obtain lighter RBDO design in the ground vehicle application.

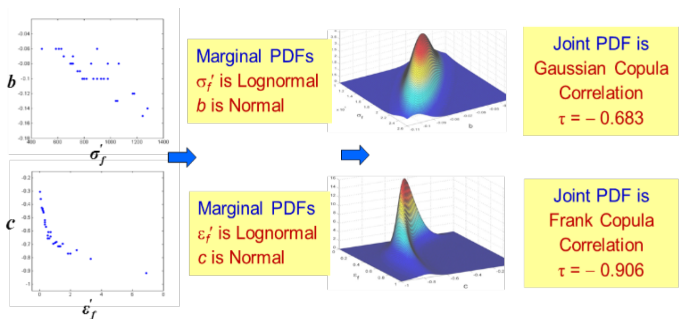



Figure 3. Input Distribution Modeling of Highly Correlated Fatigue Properties of Alloy Steel SAE 950X

## 3. SENSITIVITY-BASED RBDO METHOD

A general RBDO problem can be formulated as to

$$\begin{aligned}&\text{minimize} \quad \text{cost}\,(\mathbf{d})\\&\text{subject to}\\&P_{F_j}(\mathbf{d}) = P[G_j(\mathbf{d}(\mathbf{X})) > 0] \le P_{F_j}^{Tar} \qquad j = 1,\ldots,NC\\&\mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U,\ \ \mathbf{d} \in \mathbf{R}^{NDV},\ \text{and}\ \ \mathbf{X} \in \mathbf{R}^{NRV}\end{aligned} \tag{1}$$

where $\mathbf{d}$, $G_j$, $P_{F_j}^{Tar}$, $NC$, $NDV$, and $NRV$ are the design variable vector, $j^{\text{th}}$ constraint function, $j^{\text{th}}$ target probability of failure, numbers of constraints, design variables, and random variables, respectively.

There are two different approaches to perform RBDO: using the reliability index approach (RIA) by performing the conventional reliability analysis; and performance measure approach (PMA) given in Eq. (2)

minimize cost $(\mathbf{d})$
subject to

$$G_j(\mathbf{d}(\mathbf{X}_{MPP})) \le 0, \quad j = 1, \ldots, NC$$
$$\mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \quad \mathbf{d} \in \mathbf{R}^{NDV}, \quad \mathbf{X} \in \mathbf{R}^{NRV} \qquad (2)$$

where $\mathbf{X}_{MPP}$ is found by performing inverse reliability analysis to find the most probable point (MPP) using the first order reliability method (FORM) in the independent normalized U-space as [28-31]

$$\begin{aligned} &\text{maximize} && G_j(\mathbf{U}) \\ &\text{subject to} && \|\mathbf{U}\| = \beta_{t_j} \end{aligned} \qquad (3)$$

where $\beta_{t_j}$ is the given target reliability index.

Unlike RIA, which yields instability for some problems, PMA is shown to be robust and very accurate in identifying a probabilistic failure mode in the optimization process. While it is accurate, stable and robust, PMA is viewed to be expensive since it is a double-loop method. Thus, an enriched performance measure approach (PMA+) was developed by using the enhanced hybrid-mean value (HMV) method [7] and four computational efficiency strategy to substantially improve computational efficiency when applied to large-scale design problems [8]. In Ref. 8, PMA+ was demonstrated to be more efficient than the single-loop or serial methods.

There are two commonly used reliability analysis methods: linear approximation - FORM; and quadratic approximation - second order reliability method (SORM), of the performance functions. The reliability analysis using FORM may be accurate enough for mildly nonlinear performance functions, whereas the reliability analysis using SORM, which requires the second order sensitivity analysis, may be necessary for highly nonlinear performance functions of multi-variables. However, the second order sensitivity is rather complicate and quite expensive to obtain. Thus, a new inverse reliability analysis method [9-12], which can be used for multi-dimensional highly nonlinear systems to yield accurate failure rate calculation without requiring the second order sensitivities, was developed using the univariate dimension reduction method (DRM) [32]. Since the FORM-based reliability index ($\beta$) is inaccurate to search the most probable point (MPP) for highly nonlinear problems, a three-step computational process was developed to carry out the inverse reliability analysis: (1) constraint shift, (2) reliability index update using DRM, and (3) MPP search using the updated reliability index. Using the three steps, the new DRM-based MPP is obtained, which estimates the probability of failure of the performance function more accurately than the FORM-based MPP, while much more efficiently than SORM. The DRM-based MPP is used for the next design iteration of RBDO, which yields an accurate optimum design even for a highly nonlinear system. Since the DRM-based RBDO requires more function evaluations, the enriched performance measure approach (PMA+) with new tolerances for constraint activeness and reduced rotation matrix was used to reduce the number of function evaluations. It is noted that the DRM-based method becomes a FORM-based method if one point integration is selected.

The sensitivity-based RBDO method was extensively tested for the weight optimization with constraints on the reliability of fatigue constraints for Army ground vehicles such as M1A1 tank road-arm [33] Stryker arm [21] and HMMWV A-arm [22] components.

In carrying out RBDO of the ground vehicle components for fatigue life, it was found that correctly modeling joint distribution for correlated input variables affects the RBDO optimum design significantly. The impact of using the correct correlated input distribution is shown in the fatigue RBDO of the HMMWV front left a-arm shown in Figure 4 [22], for which the multibody dynamic analysis is carried out at TARDEC using DADS software. The fatigue RBDO for the HMMWV left front A-arm is formulated to

$$\begin{aligned} &\text{minimize} \ \ \text{Cost}(\mathbf{d}) \\ &\text{subject to} \ P[G_j(\mathbf{X}) > 0] \le P_{F_j}^{\text{Tar}}, \ \ j = 1, \cdots, 13 \\ &\qquad\qquad \mathbf{d}^{\text{L}} \le \mathbf{d} \le \mathbf{d}^{\text{U}}, \quad \mathbf{d} \in \mathbf{R}^8 \ \text{and} \ \mathbf{X} \in \mathbf{R}^{12} \end{aligned} \qquad (4)$$

where

$$\begin{aligned} &\text{Cost}(\mathbf{d}) : \text{Volume of A-arm} \\ &G_j(\mathbf{d}) = 1 - \frac{L(\mathbf{d})}{L_t}, \ \ j = 1 \sim 20 \\ &L(\mathbf{d}) : \text{Crack Initiation Fatigue Life,} \\ &L_t : \text{Target Fatigue Life (=1042 days)} \\ &P_F^{\text{Tar}} = 2.275\% \end{aligned} \qquad (5)$$

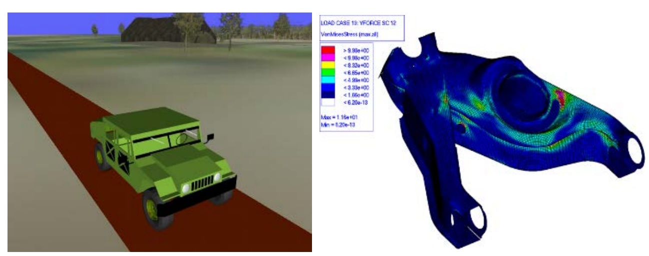


Figure 4. HMMWV Dynamics Model and Fatigue Contour of Left Front A-Arm

Eight design variables are thicknesses of plates of the A-arm, which are normally distributed with standard variations taken as 4% of the initial design variables (thicknesses) shown in Table 1; and 20 fatigue constraints (hot spots) are used. The target reliability is 2-sigma design, which means less than 2.275% probability of failure after 1042 days of continuous operation.

In addition to the eight design variables, four fatigue material properties are used as random input variables to have the total number of input variables is 12 as shown in Eq. 4. Since the HMMWV A-arm is made of SAE 950X, the correlated fatigue material property coupon test data shown in Figure 3 are used for the RBDO problem. These pairs of data can be used to simulate the fatigue lives from the strain-life relation

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\sigma_f'}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c \qquad (6)$$

where $\Delta\varepsilon/2$ is the strain amplitude, $E$ is Young's modulus and $2N_f$ is the reversals to failure. For different pairs of fatigue material property data, Eq. 6 is drawn as gray colored curves in terms of $\Delta\varepsilon$ and $N_f$ on Figures 5 (a) and (b). The red dots are the fatigue test results obtained from the coupon testing. As shown in Figure 5 (a), the gray curves do not match at all with red dots since the data is assumed not correlated; while in Figure 5 (b), the gray curves and red dots match very well since the simulation incorporates correlation. Thus, it is very important to use the correct input distribution models using copulas as shown in Figure 3.

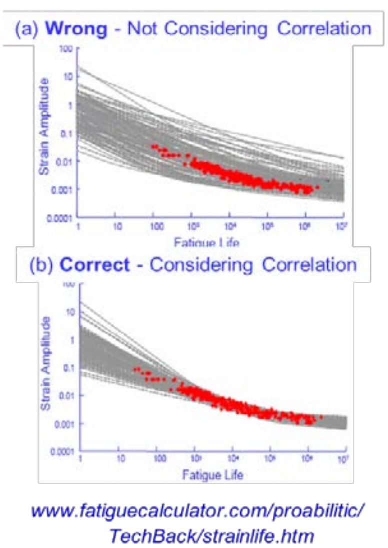



Figure 5. Fatigue Test Data of Alloy Steel SAE 950X and Simulation Results

For the HMMWV A-arm, the RBDO optimum designs are shown in Table 1 for the cases of using uncorrelated and correlated fatigue material properties. If the input fatigue material properties are correctly modelled to be correlated, then the optimum A-arm volume (which is proportional to the weight) is 157.52 in$^3$; whereas incorrectly modelled input distributions will yield the optimum A-arm volume 227.55 in$^3$, which is 45% heavier! The correlated input variable modelling using the copula in the I-RBDO software is a unique capability.

## 4. SAMPLING-BASED RBDO METHOD

While the sensitivity-based RBDO method is very effective and robust, there are many engineering design problems for which the design sensitivity information cannot be readily obtained. For these design problems, an alternative method needs to be developed. In the sampling-based RBDO, true models are approximated by using surrogate models, since direct reliability estimation using the Monte Carlo Simulation (MCS) requires evaluations at very large sample locations and evaluations for true samples are computationally expensive in most practical design applications. For this purpose the dynamic Kriging (DKG) method [14-16], which is a surrogate modeling method, and the virtual support vector machine (V-SVM) [19], which is a classification method, are developed by the Iowa team.

Table 1. RBDO Optimum Designs of HMMWV A-Arm

| | Initial Design | RBDO Optimum Designs (unit: inch) | |
|---|---|---|---|
| | | Using Uncorrelated Fatigue Property | Using Correlated Fatigue Property |
| $d_1$ | 0.1200 | 0.2926 | 0.2423 |
| $d_2$ | 0.1200 | 0.2858 | 0.1278 |
| $d_3$ | 0.1800 | 0.3418 | 0.2143 |
| $d_4$ | 0.1350 | 0.3208 | 0.2584 |
| $d_5$ | 0.2500 | 0.5852 | 0.4827 |
| $d_6$ | 0.1800 | 0.5000 | 0.5000 |
| $d_7$ | 0.1350 | 0.3278 | 0.2437 |
| $d_8$ | 0.1800 | 0.3886 | 0.1000 |
| **Volume** | **106.9** in$^3$ | **227.55** in$^3$ | **157.52** in$^3$ |

Regardless whether we use DKG or V-SVM for sampling-based RBDO, since the RBOD process is computationally expensive due to a large number of CAE analyses, for high dimensional RBDO problems, it is desirable to reduce the dimension of the RBDO problem and thus mitigate the curse-of-dimension. Therefore, it is desirable to develop an efficient and effective variable screening method [5,6] to identify important variables. In the RBDO process, the variables that induce larger output variances should be identified as important variables. An efficient approximation method based on the univariate dimension reduction method (DRM) is used to calculate output variance efficiently. To determine important variables, a hypothesis testing is used so that possible errors are contained in a user-specified error level. A required number of samples and locations are determined for calculating the output variance using the user provided error level. A quadratic interpolation method is used to calculate output variance efficiently.

Effectiveness and efficiency of the variable screening method to find important variables is demonstrated quite successfully using the Ford Motor Company's 44-dimensional multidisciplinary passenger vehicle design optimization problem for safety and NVH by selecting 18 design variables that provide the RBDO design, which is very close to the RBDO design of the 44 design variable model. Significant computational resources are saved by using the 18 design variable problem [5,6,20] for RBDO.

Another decision that we have to make is whether we should use the global domain or local window to carry out design of experiment (DoE) sampling and building DKG or V-SVM surrogate models. We are using the local window as shown in Figure 6 since RBDO requires much more accurate surrogate models than the case for DDO. The size of the local window (yellow colored hyper-cube shown in Figure 6) will be determined by dispersion (*i.e.*, standard deviations) of the given input PDFs as well as the required target reliability of the RBDO problem. If the standard deviations of the input PDFs and target reliabilities are lager, the size of the local window needs to be larger. That is, the local window size should be big enough to carry out proper reliability analysis. Once the local window is determined, we need enough number of DoE samples to obtain accurate surrogate models for reliability analysis and stochastic design sensitivity analysis using a large number of MCS samples. The rationale of the local window is to mitigate the curse-of-dimension. As shown in Figure 6, for example, if we have 2 design variables where the local window size is 1/5 of the length of each of the design variable bounds, then there are 25 (*i.e.*, 5×5) local windows. However, if we have 6 design variables, there are 15,625 (*i.e.*, $5^6$) local windows in the global domain. If 10 design variables, 9,765,625 (*i.e.*, $5^{10}$) local windows! Even if we use more than 1,000 DoE samples, these samples are spread too thin over the global domain to provide accurate surrogate models. In fact, the local window, which contains the optimum design, may not even have any sample on it, and the reliability analysis results cannot be acceptable. In our RBDO process, we need to generate surrogate models on only on certain selected local windows, which are much less than the total number of local windows on the global domain.

On the local window, hyper-sphere is used instead of the hyper-cube since too many DoE samples could be wasted in unnecessary gray area, as shown in Figure 7 (a) for the 2-D problem, for reliability analysis and design optimization for high dimensional problems. That is, the reliability analysis requires surrogate models to be accurate on the hyper-sphere. While the gray area is 21.3% of the total hyper-cube volume for the 2-D problem, for an 8-D problem, the gray area is 98.4% and DoE samples only inside the 1.6% hyper-sphere (white area) would mostly contribute to the accuracy of surrogate models. To avoid extrapolation when the surrogate models are generated, the radius $R$ of the hyper-sphere is selected to be larger (typically 1.2 times) than the target reliability index $\beta_t$ as shown in Figure 7 (a). For best utilization of DoE samples, if input variables are correlated, DoE samples on the hyper-sphere in Figure 7 (a) should be transformed using copulas as shown in Figure 7 (b).

Among surrogate modeling methods, the Kriging method has gained significant interest for its accuracy. However, in traditional Kriging methods, the mean structure is constructed using a fixed set of polynomials basis functions, correlation function type is fixed, and an optimization method is used to obtain the optimal correlation parameter which could be a local optimum. To construct an accurate Kriging model, an appropriate form of the Kriging model should be selected and the parameters should be estimated accurately.

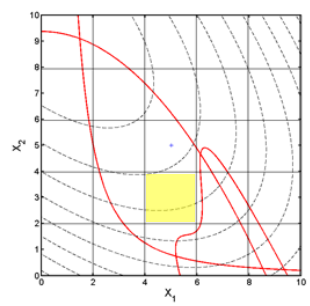
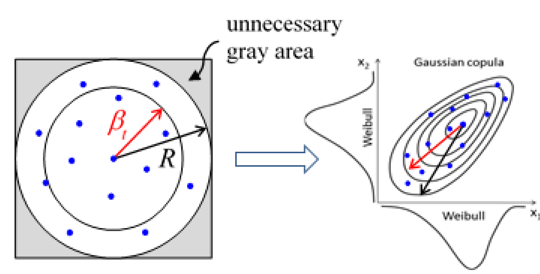


Figure 6. Local Window for DoE Sampling and Surrogate Models




(a) DoE Samples for Uncorrelated Variables

(b) DoE Samples for Correlated Variables

Figure 7. Hyper-sphere for DoE Sampling and Surrogate Models

A new DKG method is developed to fit the true model more accurately by using four methods: (1) using genetic algorithm (GA) and generalized pattern search (GPS) method for parameter search in maximum likelihood estimation (MLE), (2) using penalized MLE (PMLE) for small DoE sample size, (3) selecting the correlation model using MLE, and (4) selecting the mean structure using cross validation (CV) error.

First, the MLE technique, which is the most widely used, requires accurate correlation parameter estimation. The GPS algorithm showed better performance compared with other methods [14]. However, the performance of GPS is influenced by its initial point and thus, GA is used to provide better initial point for GPS.

Second, MLE usually provides very accurate correlation parameter values when the sample size is enough. However, MLE can be inaccurate when the sample size is small and the selected spatial correlation length (Kriging parameter) is smaller than distances between samples. In such situations, a penalized likelihood function can be used to avoid inaccurate result by using the penalty function and CV error. The penalty function has additional parameters and it can be estimated accurately using GPS. If the sample size is relatively large, the effect of the penalty function is limited. Therefore, PMLE is applied only when the sample size is small.

Third, in most applications of the Kriging method, the Gaussian correlation function is commonly used as the spatial correlation function (SCF) since it provides a relatively smooth and infinitely differentiable surface. However, there could be many different input data structures where a fixed correlation model may not be adequate enough to describe the data well. For problems with the same mean structure, MLE or PMLE can be applied to find a better correlation model.

Finally, it was shown that the accuracy of the Kriging method can be enhanced by selecting appropriate basis functions [14]. However, DKG using the process variance tends to choose the model with full basis functions, which may not be the best in terms of accuracy. In our research, leave-one-out CV is used instead of the process variance to find better mean structure.

For sampling-based RBDO, stochastic design sensitivity analysis needs to be carried out to compute the sensitivities of probabilistic constraints with respect to independent or correlated random variables and parameters [17,18]. The analytical reliability analysis involves calculation of the probability of failure, which is defined using a multi-dimensional integral as

$$\begin{aligned} P_{F_j}(\boldsymbol{\mu}) &\equiv P[\mathbf{X} \in \Omega_{F_j}] = \int_{\mathbf{R}^N} I_{\Omega_{F_j}}(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x};\boldsymbol{\mu}) d\mathbf{x} \\ &= E\left[ I_{\Omega_{F_j}}(\mathbf{X}) \right] \end{aligned} \tag{7}$$

where the mean value $\boldsymbol{\mu}$ is used as the design variable $\mathbf{d}$; $\Omega_{F_j} \equiv \left\{ \mathbf{x} : G_j(\mathbf{x}) > 0 \right\}$ is the failure set; $P[\cdot]$ represents a probability measure; $f_{\mathbf{X}}(\mathbf{x};\boldsymbol{\mu})$ is a joint PDF of the input variable $\mathbf{X}$; $E[\cdot]$ represents the expectation operator; and $I_{\Omega_{F_j}}(\mathbf{x})$ is an indicator function and defined as 1 if $\mathbf{x} \in \Omega_{F_j}$ and 0 otherwise.

The analytical stochastic design sensitivity of the probability of failure in Eq. (7) with respect to the design variable $\mu_i$ is

$$\frac{\partial P_{F_j}(\boldsymbol{\mu})}{\partial \mu_i} = \frac{\partial}{\partial \mu_i} \int_{\mathbf{R}^N} I_{\Omega_{F_j}}(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x};\boldsymbol{\mu}) d\mathbf{x} = E\left[ I_{\Omega_{F_j}}(\mathbf{X}) \frac{\partial \ln f_{\mathbf{X}}(\mathbf{X};\boldsymbol{\mu})}{\partial \mu_i} \right] \tag{8}$$

The partial derivative of the log function of the joint PDF in Eq. (8) with respect to $\mu_i$ is known as the first-order score function for $\mu_i$. Note that the score function involves derivatives of the marginal distributions and copula that is used for input uncertainty modeling of the correlated input random variables.

Using the DKG surrogate models, the score function method calculates the probability of failure and their sensitivities by applying MCS, which can be carried out efficiently using the surrogate models. It is important to note that the score function method does not require the gradients of the surrogate models, which are known to be erroneous since the surrogate model is an approximation. This behavior is similar to the fact that the stress results are not accurate while displacement results in finite element analysis (FEA) are accurate since the stresses are derivatives of approximated displacement results. Also, it is not necessary to use the transformation from *X*-space to the independent normalized *U*-space for the reliability analysis as in the case of the sensitivity-based RBDO. Since no transformation is required and the reliability is calculated in *X*-space, there is no approximation or restriction in calculating the design sensitivities of the reliability. Using the score function method, I-RBDO will allow user generated surrogate models for reliability analysis, stochastic design sensitivity analysis and RBDO, which could be a very attractive feature to some users.

It is important to note that the support vector machine (SVM) cannot provide design sensitivity of the response and thus is not useful for deterministic design optimization (DDO). On the other hand, according to Eqs. (7) and (8), (7)
the score function method requires only the classification information – a success or a failure – for MCS. Thus, SVM,

which is a classification method, can be used for RBDO by integrating with the score function method for stochastic design sensitivity analysis. Moreover, since the SVM method provides explicit limit state functions for the design constraints, it will be very efficient to estimate MCS samples, unlike the Kriging method, which is an implicit method and thus requires some computational time to estimate a very large number of MSC samples. The newly developed virtual support vector machine (V-SVM) [19] will be integrated with the score function method for RBDO in I-RBDO.

In V-SVM, virtual samples are generated near the limit state function by using an approximation method. The function values are then used for approximations of virtual samples to improve accuracy of the resulting V-SVM decision function. By introducing the virtual samples, V-SVM can overcome the deficiency in existing classification methods where only classification information is used as the input. The DKG surrogate model is used to obtain virtual samples to improve the accuracy of the decision function for highly nonlinear problems. It is demonstrated in Ref. 19 that the adaptive V-SVM is significantly more efficient in evaluation at MCS points with a very large number of test samples while maintaining a very similar level of accuracy compared to the DKG method, especially for high-dimensional problems.

For the sequential DoE sampling process, the improved constraint boundary sampling (ICBS) method is developed to insert new sequential samples near the limit state function. ICBS considers not only the distance to the limit state function but also the distance to the existing DoE samples to improve the efficiency of the sampling strategy. For parallel computation using High Performance Computing (HPC), multiple samples can be selected at once, which makes the sequential DoE sampling process more efficient.

In some existing commercial softwares, a large number of options for surrogate modeling methods are provided for RBDO, which is not necessarily beneficial to users unless the software informs the user which surrogate model is the best for the problem the user is solving. It is desirable to provide a flagship surrogate modeling or classification method that is adaptive, robust, accurate for RBDO and easy to use. As for the flagship surrogate modeling method, I-RBDO uses our DKG surrogate models, where 21 different Kriging models (3 basis functions and 7 correlation models, plus optimum correlation parameters determined) are implemented, and I-RBDO selects the best Kriging model systematically for user's RBDO problem. It is very well possible that different Kriging models may be selected at different local windows during RBDO iterations. Between Kriging and V-SVM, I-RBDO provides plenty options for surrogate models for RBDO.

The sampling-based RBDO part of I-RBDO is shown in Figure 8. This software can be integrated with any CAE softwares as black box tools as shown in Figure 8. The interface can be simple ASCII data.

## 5. COLLABORATIVE EVALUATION OF I-RBDO SOFTWARE

The research software I-RBDO has been provided to selected academia, industry, TARDEC HPC and ARL DSP for evaluation of its effectiveness by applying to various test problems. The collaborating teams integrated I-RBDO with their CAE simulation softwares for applications. I-RBDO has been successfully applied to optimize designs of passenger vehicles for safety and NVH [20]; durability [21,22]; casting process design [23]; ship hydrodynamics; fluid-structure interaction [24]; welding design [25]; superconducting magnetic energy storage system [26]; electro-thermal polysilicon actuator [27]; etc. Interested readers are referred to above cited papers for detailed information on successful applications of I-RBDO.

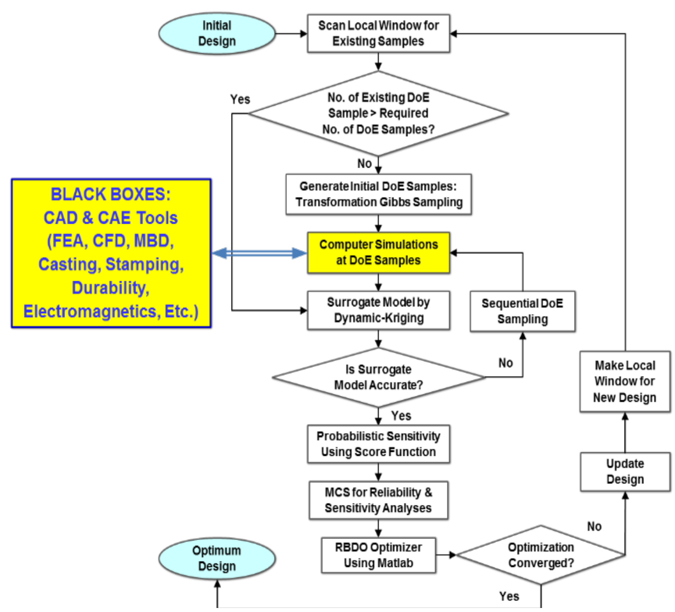



Figure 8. Sampling-Based RBDO with Simulation Softwares as Black-Boxes

In this paper, an application of I-RBDO to the Ford passenger vehicle for safety and NVH [5,6,20] is presented. The problem includes passenger safety under full frontal and 40% offset frontal impacts; as well as NVH as design constraints. There are a total of 11 performance measures: nine safety and two NVH measures as shown in Table 2.

The 44 random variables shown in Table 3 represent thicknesses of the vehicle body. All statistically independent random variables follow normal distribution. The design variable vector **d** (given at the baseline design values in *mm* in Table 3) is the mean vector of the 44 random variables, and there is no random parameter in this problem. Among those random variables, six random variables ($X_1$ to $X_5$ and $X_8$) are common variables for both the safety and NVH measures, two ($X_6$ and $X_7$) are variables only for the safety measures, and the other 36 random variables are only for the NVH measures.

This problem requires 3.5 hours for the impact dynamic analysis for safety and the modal analysis for NVH. Thus, actual analyses would take too long to carry out various testing of the proposed method thoroughly. Ford Motor Company provided 44-D surrogate models on the global domain between the lower bound $\mathbf{d}^L$ and upper bound $\mathbf{d}^U$.

The purpose of this study is to demonstrate three key capabilities of the I-RBDO software: (1) robustness and efficiency of the RBDO algorithm using the Ford 44-D surrogate models, (2) accuracy of DKG surrogate models for RBDO, and (3) effectiveness of the variable screening method. The RBDO is formulated to

$$\begin{aligned} &\text{minimize } \mathit{Weight}(\mathbf{d}) \\ &\text{subject to } \begin{array}{l} P\left[G_i(\mathbf{X}) > \text{Baseline}_i\right] \le 10\%, \quad i = 1,\ldots,11 \\ \mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \quad \mathbf{d} \in \mathbb{R}^{NDV}, \quad \text{and} \quad \mathbf{X} \in \mathbb{R}^{NRV} \end{array} \end{aligned} \tag{9}$$

Table 2. Performance Measures

| Mode | | Function | Value | Feasibility Decision |
|---|---|---|---|---|
| Safety | Full Frontal Impact | $G_1$ | Chest G | $\le Baseline_i$ |
| | | $G_2$ | Crush Displacement | |
| | 40% Offset Impact | $G_3$ | Brake Pedal | |
| | | $G_4$ | Footrest | |
| | | $G_5$ | Left Toepan | |
| | | $G_6$ | Center Toepan | |
| | | $G_7$ | Right Toepan | |
| | | $G_8$ | Left IP | |
| | | $G_9$ | Right IP | |
| NVH | | $G_{10}$ | Torsion Mode | |
| | | $G_{11}$ | Vertical Bending mode | |

For the first capability testing, the RBDO problem is solved using the Ford 44-D surrogate models (*i.e.*, $NDV = NRV = 44$). Table 4 shows the baseline design, DDO and RBDO optimum designs obtained using I-RBDO and NSGA-II software. Column 1 shows 11 constraints. Columns 2-4 show the weights at the baseline design, DDO design obtained using I-RBDO, and deterministic design obtained using NSGA-II after 100 iterations, respectively. A number of constraints at these three designs yield around 50% probability of failures. Columns 5-7 show RBDO designs obtained using I-RBDO starting at three designs shown at columns 2-4, respectively. It is interesting to note that, starting from the three different initial designs, three RBDO optimum processes converged practically to the same design (weight) with very close reliabilities for 11 design constraints. This demonstrates robustness and efficiency of the RBDO algorithm in I-RBDO due the accurate stochastic design sensitivity analysis using the score functions. So if the user has accurate surrogate models, I-RBDO will provide good optimum designs.

Table 3. Input Random Variables and Baseline Design

| RVs | Dist. Type | **d** | STDV | $\mathbf{d}^L$ | $\mathbf{d}^U$ |
|---|---|---|---|---|---|
| $X_1$ | Normal | 1.9 | 0.05 | 1.5 | 2.3 |
| $X_2$ | Normal | 1.91 | 0.05 | 1.5 | 2.3 |
| $X_3$ | Normal | 2.51 | 0.06 | 2.0 | 3.0 |
| $X_4$ | Normal | 2.4 | 0.06 | 1.9 | 2.9 |
| $X_5$ | Normal | 2.55 | 0.06 | 2.0 | 3.1 |
| $X_6$ | Normal | 2.25 | 0.06 | 1.8 | 2.7 |
| $X_7$ | Normal | 2.25 | 0.06 | 1.8 | 2.7 |
| $X_8$ | Normal | 1.5 | 0.03 | 1.2 | 1.8 |
| $X_{10}$ | Normal | 1.28 | 0.03 | 0.9 | 1.6 |
| $X_{11}$ | Normal | 1.4 | 0.03 | 1.0 | 1.8 |
| $X_{12}$ | Normal | 1.1 | 0.03 | 0.8 | 1.4 |
| $X_{13}$ | Normal | 2.2 | 0.06 | 1.7 | 2.7 |
| $X_{14}$ | Normal | 1.5 | 0.03 | 1.2 | 1.8 |
| $X_{15}$ | Normal | 1.25 | 0.03 | 0.9 | 1.6 |
| $X_{16}$ | Normal | 2.5 | 0.06 | 2.0 | 3.0 |
| $X_{17}$ | Normal | 2.0 | 0.05 | 1.5 | 2.5 |
| $X_{18}$ | Normal | 1.4 | 0.03 | 1.1 | 1.7 |
| $X_{20}$ | Normal | 1.22 | 0.03 | 0.9 | 1.5 |
| $X_{23}$ | Normal | 0.75 | 0.03 | 0.6 | 1.0 |
| $X_{24}$ | Normal | 1.9 | 0.05 | 1.5 | 2.3 |
| $X_{25}$ | Normal | 0.65 | 0.03 | 0.5 | 0.8 |
| $X_{26}$ | Normal | 0.85 | 0.03 | 0.6 | 1.1 |
| $X_{27}$ | Normal | 0.85 | 0.03 | 0.6 | 1.1 |
| $XN_1$ | Normal | 0.9 | 0.03 | 0.7 | 1.1 |
| $XN_2$ | Normal | 1.1 | 0.03 | 0.8 | 1.4 |
| $XN_3$ | Normal | 1.55 | 0.05 | 1.2 | 1.9 |
| $XN_4$ | Normal | 0.9 | 0.03 | 0.7 | 1.1 |
| $XN_5$ | Normal | 1.5 | 0.03 | 1.2 | 1.8 |
| $XN_6$ | Normal | 1.2 | 0.03 | 0.9 | 1.5 |
| $XN_7$ | Normal | 1.1 | 0.03 | 0.8 | 1.4 |
| $XN_8$ | Normal | 1.52 | 0.05 | 1.2 | 1.9 |
| $XN_9$ | Normal | 0.8 | 0.03 | 0.6 | 1.0 |
| $XN_{10}$ | Normal | 0.8 | 0.03 | 0.6 | 1.0 |
| $XN_{11}$ | Normal | 1.2 | 0.03 | 0.9 | 1.5 |
| $XN_{12}$ | Normal | 0.75 | 0.03 | 0.6 | 0.9 |
| $XN_{13}$ | Normal | 0.75 | 0.03 | 0.6 | 0.9 |
| $XN_{14}$ | Normal | 0.75 | 0.03 | 0.6 | 0.9 |
| $XN_{15}$ | Normal | 1.0 | 0.03 | 0.8 | 1.2 |
| $XN_{16}$ | Normal | 1.14 | 0.03 | 0.9 | 1.4 |
| $XN_{17}$ | Normal | 1.2 | 0.03 | 0.9 | 1.5 |
| $XN_{18}$ | Normal | 1.4 | 0.03 | 1.1 | 1.7 |
| $XN_{19}$ | Normal | 1.2 | 0.03 | 0.9 | 1.5 |
| $XN_{20}$ | Normal | 1.4 | 0.03 | 1.1 | 1.7 |
| $XN_{21}$ | Normal | 2.13 | 0.06 | 1.7 | 2.6 |

The last column of Table 4 shows the optimum design obtained using NSGA-II RBDO after it took more than 100 hours to run with 60,000 DoE simulations as NSGA-II uses a genetic algorithm. All reliability analysis results in this table are obtained by carrying out MCS using the Ford 44-D surrogate models.

Even though I-RBDO can very effectively find the RBDO optimum design using Ford 44-D surrogate models as shown in Table 4, the optimum design may not be useful for design guidance in practical automotive application since the 44-D surrogate models are not accurate enough, especially for assessment of reliability of the optimum design. In fact, 44 design variables are too many to obtain accurate surrogate models due to the curse-of-dimension, especially on the global domain, no matter what surrogate modeling methods are used. To be useful, it is necessary to have an accurate surrogate modeling capability that can be confidently used for design guidance in practical applications. For this, as described before, we propose to use the local window concept. Even then, 44-D surrogate models are not easy to obtain accurately. That is the reason I-RBDO is providing the variable screening method.

Table 4. Robustness of I-RBDO for the 44-D Problem

| Objective, Constraints, etc. | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Initial Designs | | | RBDO Final Designs Using I-RBDO | | | NSGA-II |
| | Baseline | DDO | NSGA-II* | Starting from Baseline | Starting from DDO | Starting from NSGA-II* | Starting from NSGA-II** |
| Weight | 269.47 | 222.91 | 240.12 | 225.68 | 225.66 | 225.67 | 227.44 |
| | | | | | | | |
| G1 | 48.22% | 49.61% | 32.95% | 10.05% | 10.07% | 9.96% | 9.53% |
| G2 | 51.48% | 51.44% | 49.57% | 10.09% | 10.18% | 10.11% | 11.47% |
| G3 | 54.15% | 57.18% | 0.01% | 0.00% | 0.00% | 0.00% | 0.00% |
| G4 | 55.57% | 37.65% | 0.01% | 0.12% | 0.09% | 0.10% | 0.04% |
| G5 | 58.96% | 4.38% | 0.59% | 1.91% | 1.82% | 1.84% | 1.20% |
| G6 | 59.71% | 24.55% | 2.52% | 10.00% | 10.03% | 9.92% | 7.75% |
| G7 | 59.92% | 61.26% | 19.05% | 10.06% | 9.99% | 9.89% | 8.56% |
| G8 | 53.19% | 0.12% | 13.79% | 9.14% | 9.91% | 10.04% | 9.46% |
| G9 | 51.17% | 51.90% | 38.44% | 9.96% | 9.91% | 9.95% | 4.08% |
| G10 | 49.05% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% |
| G11 | 52.46% | 52.24% | 43.80% | 10.05% | 9.94% | 10.08% | 7.55% |
| | | | | | | | |
| Terminal Cond. | | | | 1.00E-03 | 1.00E-03 | 1.00E-03 | |
| Computation Time (h) | | | | 6 | 54 | 3 | >100 |
| # of Design Iterations | | | | 29 | 43 | 19 | |

* Selected arbitrarily from deterministic NSGA-II run after 100 iterations
** 300 generations×200 points/generation = **60,000**
DDO: deterministic design optimization obtained using I-RBDO

To demonstrate the second and third capabilities of I-RBDO described above, from now on, we will refer the Ford 44-D surrogate models as the "benchmark" models. In this way, even though we are not using the actual DoE samples with impact dynamic analysis for safety and the modal analysis for NVH, we can verify I-RBDO capabilities thoroughly.

For the second study, a reduced number of design variables ($NDV$ = $NRV$ = 14) are selected to obtain accurate DKG surrogate models and compare it with the Ford 44-D benchmark models. The 14 design variables, $X_1$, $X_2$, $X_3$, $X_4$, $X_5$, $X_6$, $X_7$, $X_8$, $X_{10}$, $X_{20}$, $X_{23}$, $X_{25}$, $X_{26}$, and $XN_1$, are selected using the variable screening method. These design variables are selected based on their contributions to the outcome variances of the 11 design constraints. For this study, the DKG surrogate models are generated using I-RBDO where the DoE samples are obtained from the Ford 44-D benchmark models. Using the DKG surrogate models and starting from the initial baseline design, the RBDO optimum design is obtained as shown in column 3 in Table 5. To validate accuracy of the DKG surrogate models, RBDO is carried out using the Ford 44-D benchmark models, but using only selected 14 design variables, as shown in column 4 in Table 5. These two RBDO optimum designs have identical weights (259.83) and reliability constraint results are practically the same. This demonstrates accuracy of the DKG surrogate models. Further, two RBDO optimum designs shown in Table 6 are the same. This clearly demonstrates accuracy of the DKG surrogate models.

Table 5. 14-D RBDO Optimum Costs and Constraint Reliabilities Using I-RBDO and Ford Benchmark

| Performance Measure | Baseline Design | Using I-RBDO DKG | Using Ford Benchmark |
|---|---|---|---|
| Weight | 269.47 | 259.83 | 259.83 |
| $G_1$ | 48.22% | 9.94% | 9.94% |
| $G_2$ | 51.48% | 10.11% | 10.02% |
| $G_3$ | 54.15% | 0.00% | 0.00% |
| $G_4$ | 55.57% | 0.11% | 0.12% |
| $G_5$ | 58.96% | 1.95% | 1.91% |
| $G_6$ | 59.71% | 10.01% | 10.02% |
| $G_7$ | 59.92% | 9.93% | 10.03% |
| $G_8$ | 53.19% | 9.98% | 9.97% |
| $G_9$ | 51.17% | 9.99% | 9.98% |
| $G_{10}$ | 49.05% | 0.00% | 0.00% |
| $G_{11}$ | 52.46% | 9.97% | 10.02% |
| No. of Des. Iter. | | 30 | 17 |

Table 6. 14-D RBDO Optimum Designs Using I-RBDO and Ford Benchmark

| Design Variables | Using I-RBDO DKG | Using Ford Benchmark |
|---|---|---|
| $X_1$ | 1.8366 | 1.8359 |
| $X_2$ | 2.1804 | 2.1807 |
| $X_3$ | 2.8561 | 2.8576 |
| $X_4$ | 1.9810 | 1.9851 |
| $X_5$ | 2.7228 | 2.7233 |
| $X_6$ | 2.2497 | 2.2501 |
| $X_7$ | 2.3185 | 2.3169 |
| $X_8$ | 1.7966 | 1.7985 |
| $X_{10}$ | 0.9 | 0.9 |
| $X_{20}$ | 0.9 | 0.9 |
| $X_{23}$ | 0.6 | 0.6 |
| $X_{25}$ | 0.5429 | 0.5424 |
| $X_{26}$ | 1.1 | 1.1 |
| $XN_1$ | 0.7 | 0.7 |

On the other hand, as shown in Table 5, the optimum weights of the 14-D models (259.83) are higher than the optimum weights of the Ford 44-D benchmark models obtained in Table 4 (225.68, 225.66, and 225.67) since the design space is smaller with only 14 design variables. Also, the selected 14 design variables are entirely based on the 11 design constraints and without consideration of the cost function (*i.e.*, weight). Thus, four additional design variables, $XN_4$, $XN_9$, $XN_{10}$, and $XN_{11}$, are selected from remaining 30 design variables based on the highest sensitivity for the cost function. The 18-D RBDO optimum design is shown in Table 7. As shown in Table 7, the optimum weight is reduce significantly to 244.17, while the reliability constraints are satisfied. Two RBDO optimum designs have identical weights (244.17, 244.18) and reliability constraint results are practically the same. The RBDO optimum designs are shown in Table 8, which shows that they are the same design. Again, this clearly demonstrates accuracy of the DKG surrogate models, even though RBDO requires much more accurate surrogate models than the DDO application.

Table 7. 18-D RBDO Optimum Costs and Constraint Reliabilities Using I-RBDO and Ford Benchmark

| Performance Measure | Baseline Design | Using I-RBDO DKG | Using Ford Benchmark |
|---|---|---|---|
| Cost | 269.47 | 244.17 | 244.18 |
| $G_1$ | 48.22% | 9.96% | 10.01% |
| $G_2$ | 51.48% | 10.04% | 10.02% |
| $G_3$ | 54.15% | 0.00% | 0.00% |
| $G_4$ | 55.57% | 0.09% | 0.11% |
| $G_5$ | 58.96% | 2.04% | 1.85% |
| $G_6$ | 59.71% | 10.02% | 9.97% |
| $G_7$ | 59.92% | 9.95% | 9.94% |
| $G_8$ | 53.19% | 9.91% | 10.00% |
| $G_9$ | 51.17% | 9.87% | 10.01% |
| $G_{10}$ | 49.05% | 0.00% | 0.00% |
| $G_{11}$ | 52.46% | 9.93% | 9.97% |
| No. of Des. Iter. | | 20 | 20 |

While the reliability constraints are very well satisfied by the 14-D and 18-D RBDO optimum designs as shown in Tables 5 and 7, respectively, these reliability analyses were carried out using 14-D and 18-D DKG surrogate models or 14-design and 18-design variables in the Ford 44-D benchmark models, respectively. However, the design variables that were not selected for RBDO are random variables and they are fixed at their mean values at the baseline design point. This raises a question that whether the variable screening method selected proper variables so that 14-D or 18-D RBDO optimum designs satisfy the reliability constraints when we consider all 44 variables to be random, which is the real situation. Table 9 shows the reliability analysis results of these optimum designs using the Ford full 44-D benchmark models. The results in Table 9 are quite close to the results in Tables 5 and 7, respectively, except for the constraint $G_{11}$. Thus, the variable screening method is effective to control all constraints to satisfy the target reliability of 10%, except for the constraint $G_{11}$, which are slightly off. Note that the reliability assessments (11.23% and 11.27%) for both RBDO optimum designs in Table 9 are true as they are calculated using the Ford 44-D benchmark models. Comparing the results for $G_{11}$ in Table 9 and the corresponding results in Tables 5 and 7, respectively, the reliability differences are 11.23–9.97=1.26% and 11.17–9.93=1.24%, which can be reduced by selecting more variables in the variable screening method. This will be a trade-off issue between the computer resources and desired accuracy. Thus, effectiveness and efficiency of the variable screening method is demonstrated quite successfully. The local window method and variable screening method in I-RBDO are the most viable option to overcome the curse-of-dimension.

Table 8. 18-D RBDO Optimum Designs Using I-RBDO and Ford Benchmark

| Design Variables | Using I-RBDO DKG | Using Ford Benchmark |
|---|---|---|
| $X_1$ | 1.8336 | 1.8425 |
| $X_2$ | 2.1806 | 2.1771 |
| $X_3$ | 2.8540 | 2.8654 |
| $X_4$ | 1.9856 | 1.9525 |
| $X_5$ | 2.7261 | 2.7209 |
| $X_6$ | 2.2558 | 2.2464 |
| $X_7$ | 2.3207 | 2.3265 |
| $X_8$ | 1.786 | 1.8 |
| $X_{10}$ | 0.9 | 0.9 |
| $X_{20}$ | 0.9 | 0.9 |
| $X_{23}$ | 0.6 | 0.6 |
| $X_{25}$ | 0.5871 | 0.5826 |
| $X_{26}$ | 1.1 | 1.1 |
| $XN_1$ | 0.7 | 0.7 |
| $XN_4$ | 0.7 | 0.7 |
| $XN_9$ | 0.6 | 0.6 |
| $XN_{10}$ | 0.6 | 0.6 |
| $XN_{11}$ | 0.9 | 0.9 |

## 6. COMMERCIALIZATION PLAN

With successful applications of I-RBDO, the Iowa team established a small start-up company RAMDO Solutions, LLC to develop a commercial software Reliability Analysis & Multidisciplinary Design Optimization (RAMDO). For this purpose, the research software I-RBDO was delivered to RAMDO Solutions, LLC. For commercialization, the company was awarded Iowa GAP funding and successfully obtained Iowa State LAUNCH Loan in 2014. In May, 2014, the company was awarded an Army SBIR Phase I funding. This commercialization of the ARC funded I-RBDO software represents a major technology transfer for the benefit of the Army. The success of TARDEC led research and development is highlighted by taking a significant multi-year project like this and demonstrating a full transition of the technology to the commercial marketplace. Since RAMDO is a computational software for Multidisciplinary RBDO, it will be integrated with PIDO systems. RAMDO Solutions, LLC needs to work major PIDO software companies to develop partnership.

Table 9. Reliability Analysis Results Using Ford 44-D Benchmark Models

| Performance Measure | Variable Screening 14-D | Variable Screening 18-D |
|---|---|---|
| Cost | 259.83 | 244.17 |
| $G_1$ | 9.96% | 10.00% |
| $G_2$ | 10.11% | 10.04% |
| $G_3$ | 0.00% | 0.00% |
| $G_4$ | 0.12% | 0.09% |
| $G_5$ | 1.93% | 1.98% |
| $G_6$ | 10.05% | 10.05% |
| $G_7$ | 10.04% | 9.91% |
| $G_8$ | 10.03% | 9.97% |
| $G_9$ | 9.96% | 9.96% |
| $G_{10}$ | 0.00% | 0.00% |
| $G_{11}$ | 11.23% | 11.17% |

## 7. ACKNOWLEDGEMENT

The authors wish to acknowledge the technical and financial support of the Automotive Research Center (ARC) in accordance with Cooperative Agreement W56HZV-04-2-0001 U.S. Army Tank Automotive Research, Development and Engineering Center (TARDEC), Warren, MI.

The authors sincerely appreciate Dr. Ren-Jye Yang and his team at Ford Motor Company for providing their 44-D surrogate models of the safety and NVH problem for our study and collaborating to verify I-RBDO capabilities.



## 8. REFERENCES

[1] Noh, Y., Choi, K.K., and Du, L., "New Transformation of Dependent Input Variables Using Copula for RBDO," *7th World Congress on Structural and Multidisciplinary Optimization*, May 21-25, 2007, Seoul, Korea, ***Runner-up paper of the ISSMO-Springer Prize 2007***.

[2] Noh, Y., Choi, K.K., and Du, L., "Reliability-based design optimization of problems with correlated input variables using a Gaussian Copula," *Structural and Multidisciplinary Optimization*, Vol. 38, No. 1, 2009, pp. 1–16.

[3] Noh, Y., Choi, K.K., and Lee, I., "Identification of Marginal and Joint CDFs Using the Bayesian Method for RBDO," *Structural and Multidisciplinary Optimization*, Vol. 40, No. 1, 2010, pp. 35-51.

[4] Noh, Y., Choi, K.K., and Lee, I., "Comparison Study between MCMC-based and Weight-based Bayesian Methods for Identification of Joint Distribution," *Structural and Multidisciplinary Optimization*, Vol. 42, 2010, pp. 823–833.

[5] Cho, H., Bae, S., Choi, K.K., Lamb, D., and Yang, R-J., "An Efficient Variable Screening Method for Effective Surrogate Models for Reliability-Based Design Optimization" *10th World Congress on Structural and Multidisciplinary Optimization*, May 20-24, 2013, Orlando, Florida.

[6] Cho, H., Bae, S., Choi, K.K., Lamb, D., and Yang, R-J., "An Efficient Variable Screening Method for Effective Surrogate Models for Reliability-Based Design Optimization," *Structural and Multidisciplinary Optimization*, DOI 10.1007/s00158-014-1096-9, 2014.

[7] Youn, B.D., Choi, K.K., and Du, L., "Adaptive Probability Analysis Using An Enhanced Hybrid Mean Value Method," *Structural and Multidisciplinary Optimization*, Vol. 29, No. 2, 2005, pp. 134-148.

[8] Youn, B.D., Choi, K.K., and Du, L., "Enriched Performance Measure Approach for Reliability-Based Design Optimization," *AIAA Journal*, Vol. 43, No. 4, 2005, pp. 874-884.

[9] Lee, I., Choi, K.K., Du, L., and Gorsich, D. "Dimension Reduction Method (DRM) Based RBDO for Highly Nonlinear Systems" *7th World Congress on Structural and Multidisciplinary Optimization*, May 21-25, 2007, Seoul, Korea, ***Received the ISSMO-Springer Prize 2007***.

[10] Lee, I., Choi, K.K., Du, L., and Gorsich, D., "Dimension Reduction Method for Reliability-Based Robust Design Optimization," *Special Issue of Computer & Structures: Structural and Multidisciplinary Optimization*, Vol. 86, 2008, pp. 1550–1562.

[11] Lee, I., Choi, K.K., Du, L., and Gorsich, D., "Inverse Analysis Method Using MPP-based Dimension Reduction for Reliability-Based Design Optimization of Nonlinear and Multi-Dimensional Systems," *Special Issue on Computational Methods in Optimization Considering Uncertainties, Comp. Meth. Appl. Mech. Engrg.*, Vol. 198, No. 1, 2008, pp. 14-27.

[12] Lee, I., Choi, K.K., and Gorsich, D., "Sensitivity Analyses of FORM-Based and DRM-Based

Performance Measure Approach (PMA) for Reliability-Based Design Optimization (RBDO)," *Int. J. of Numerical Methods in Engineering*, Volume 82, Issue 1, 2010, pp. 26–46.

[13] Lee, I., Choi, K.K., and Gorsich, D., "System Reliability-based Design Optimization Using MPP-based Dimension Reduction Method," *Structural and Multidisciplinary Optimization*, Vol. 41, No. 6, 2010, pp. 823-839.

[14] Zhao, L., Choi, K.K., and Lee, I., "Metamodeling Method Using Dynamic Kriging for Design Optimization," *AIAA Journal*, Vol. 49, No. 9, 2011, pp. 2034-2046.

[15] Zhao, L., Choi, K.K., Lee, I., and Gorsich, D., "Conservative Surrogate Model using Weighted Kriging Variance for Sampling-based RBDO," *ASME Journal of Mechanical Design*, Vol. 135, 091003, 2013.

[16] Song, H., Choi, K.K., Lamb, D., "A Study on Improving Accuracy of Kriging Models by Using Correlation Model Selection and Penalized Log-Likelihood Function," *10th World Congress on Structural and Multidisciplinary Optimization*, May 20-24, 2013, Orlando, Florida.

[17] Lee, I., Choi, K.K., Noh, Y., Zhao, L., and Gorsich, D., "Sampling-Based Stochastic Sensitivity Analysis Using Score Functions for RBDO Problems with Correlated Random Variables," *ASME Journal of Mechanical Design*, Vol.133, 021003, 2011.

[18] Lee, I., Choi, K.K., and Zhao, L., "Sampling-Based RBDO Using the Stochastic Sensitivity Analysis and Dynamic Kriging Method," *Structural and Multidisciplinary Optimization*, Vol. 44, 2011, pp. 299-317.

[19] Song, H., Choi, K.K., Lee, I., Zhao, L., and Lamb, D., "Adaptive Virtual Support Vector Machine for Reliability Analysis of High-Dimensional Problems," *Structural and Multidisciplinary Optimization*, Volume 47, Issue 4, 2013, pp. 479-491.

[20] Choi, K.K., Gaul, N., Song, H., Cho, H., Lee, I., Zhao, L., Noh, Y., Lamb, D., Sheng, J., Gorsich, D., Yang, R-J., Shi, L., Epureanu, B., and Hong, H-S., "ARC Tech Transfer - Iowa Reliability-Based Design Optimization (I-RBDO) Code and Its Applications" *Thrust Area 3 Case Study for the 2012 ARC Conference*, May 21, 2012, Ann Arbor, MI.

[21] Choi, K.K. and Lamb, D., "Reliability Analysis and Design Optimization of Military Ground Vehicles Using TARDEC High Performance Computing" *Thrust Area 3 Case Study for the 2007 ARC Conference*, May 15, 2007, Ann Arbor, MI.

[22] Choi, K.K. and Lamb, D., "Reliability-Based Design Optimization with Associated Confidence Levels under Input Model Uncertainties for Ground Vehicle Weight Reduction" *Thrust Area 3 Case Study for the 2009 ARC Conference*, May 11, 2009, Ann Arbor, MI.

[23] Hardin, R. Choi, K.K., Beckermann, C., "Reliability-Based Casting Process Design Optimization," *2014 TMS Annual Meeting & Exhibition*, San Diego Convention Center, CA, February 16-20, 2014.

[24] Jang, H-R., Cho, S., and Choi, K.K., "Sampling-based RBDO of Fluid-Solid Interaction (FSI) Problems," *IMechE-C; Journal of Mechanical Engineering Science*, 0(0) 1–19, DOI: 10.1177/0954406213509762, 2013.

[25] Choi, M., Cho, S., Choi, K.K., and Cho, H., "Sampling-based RBDO of Ship Hull Structures Considering Thermo-elasto-plastic Residual Deformation," *Mechanics Based Design of Structures and Machines*, to appear, 2014.

[26] Kim, D-W., Choi, N-S., Choi, K.K., and, Kim, D-H., "Optimization of a SMES Magnet in Due Consideration of Uncertainty Utilizing Sampling-based Reliability Analysis," *23rd International Conference on Magnet Technology*, Boston, MA, July 14-19, 2013.

[27] Kim, D-W., Choi, N-S., Choi, K.K. and Kim, D-H., "Sequential Design Method for Geometric Optimization of an Electro-Thermal Microactuator based on Dynamic Kriging Models," *CEFC 2014*, Annecy, France, May 25-28, 2014.

[28] Tu, J., Choi, K.K., Park, Y.H., "A New Study on Reliability Based Design Optimization," *ASME Journal of Mechanical Design*, Vol. 121, No. 4, 1999, pp. 557-564.

[29] Choi, K.K. and Youn, B.D., "Hybrid Analysis Method for Reliability-Based Design Optimization," *27th ASME Design Automation Conference*, September 9-12, 2001, Pittsburgh, PA. ***Received the 2001 ASME Black and Decker Best Paper Award***.

[30] Youn, B.D., Choi, K.K., and Park, Y.H., "Hybrid Analysis Method for Reliability-Based Design Optimization," *ASME Journal of Mechanical Design*, Vol. 125, No. 2, 2003, pp. 221-232.

[31] Youn, B.D., Choi, K.K., and Yang, R.J., "Efficient Evaluation Approaches for Probabilistic Constraints in Reliability-Based Design Optimization," *5th World Congress of Structural and Multidisciplinary Optimization*, May 19-23, 2003, Lido di Jesolo-Venice, Italy. ***Received the ISSMO-Springer Prize 2003***.

[32] Rahman, S., "Stochastic Sensitivity Analysis by Dimensional Decomposition and Score Functions," Probabilistic Engineering Mechanics, Vol. 24, 2009, pp. 278-287.

[33] Lamb, D., Gorsich, D., Krayterman, D., Choi, K., Hardee, E., Du, L., Youn, B., Bettig, B., and Ghiocel, D., "System Level RBDO for Military Ground

Vehicles using High Performance Computing," *2008 SAE World Congress*, April 14-18, 2008, Paper #2008 01-0543, Detroit, MI.