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MODULAR EXHAUST DESIGN AND MANUFACTURING TECHNIQUES FOR LOW COST MID VOLUME RAPID BUILD TO ORDER SYSTEMS

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ABSTRACT

We propose new methods to help automate the design of customizable mufflers, as well as modular manufacturing techniques targeted at mid volume quantities. A successful solution would reduce the price point of a muffler to an estimated $500 per unit for a order size between 10 and 1000 units. In the ideal case, customers would not need to inventory mufflers because lead times would be fast and managed.

INTRODUCTION

Managing the acoustic signature of military vehicles has become an increasing priority for the U.S. Armed Forces and can play a critical role in the safety of soldiers and the tactical strategy for mission planning. The management of low frequency sound must be paid special attention because low frequency sounds propagate effectively through the atmosphere resulting in unacceptable acoustic vehicle detection ranges, and because treating low frequency noises requires relatively large silencer structures which are expensive to manufacture and integrate. Currently, signature requirements are met by developing a custom muffler for each military project which is then hand-assembled using low volume prototyping manufacturing techniques. This method results in significant engineering and manufacturing time which translates into longer project lead times and cost.

An alternative method for the design and manufacture of low cost performance mufflers is discussed in this paper. The price point of the muffler systems are reduced by attacking the two most costly components in the old method, which are engineering design time and building custom tooling and fixtures. In the proposed method, a computer algorithm is used to automate the design iterations of the exhaust system, reducing the engineering overhead from weeks to days. Custom tooling and fixture costs are eliminated by limiting the available muffler components to pieces easily fabricated using only general manufacturing techniques.

This paper focuses on the automated design of muffler systems, though the general manufacturing techniques are discussed at the end of the paper.

First, a theory is developed for quickly and accurately predicting the performance a muffler system. Muffler components are modeled as 1 dimensional elements represented by a series of 4 coefficient matrices known as transfer matrices. It will be shown that muffler performance (both acoustic and back pressure) can be quickly and accurately estimated using this theory. This model can be quickly iterated over to develop a customized muffler for a given set of performance requirements.

The acoustic performance model is then transformed into a manufacturing model which is used in the fabrication of the muffler system. The transformation only allows for physical components to be used which can be fabricated from a process which does not require custom tooling or fixtures.

THEORY OF ACOUSTIC FILTERS

For the purposes of analysis, muffler systems may be broken up into acoustic elements which can be represented using one dimensional acoustic circuit analogs. These analogs may then be connected into a circuit graph and the graph may be solved to predict the performance characteristics of the muffler. In the case of exhaust systems, the performance characteristics are usually composed of two separate components, known as transmission loss and back pressure. Transmission loss is important for calculating the noise reduction of the muffler against frequency. Backpressure is an important engine performance characteristic as it describes how much engine power will be lost pushing exhaust through the muffler.

To generate a circuit equivalent for a muffler system, first principals must be established. The following is a table which describes the analogous terms between electrical and acoustic circuit components. These analogs can be used to adapt powerful tools from the field of electrical engineering, like lumped element analysis, for use in the acoustic domain. [1]

Acoustical Terms			**Electrical Terms**		
Variable		**Units**	**Variable**		**Units**
Pressure	p	$\frac{N}{M^2}$	Voltage	e	Volts
Mass Velocity	v	$\frac{Kg}{s}$	Current	i	Amperes
Acoustic Impedance	Z	$\frac{1}{ms}$	Electrical Impedance	Z	Ohms
Resistance	R	$\frac{1}{ms}$	Resistance	R	Ohms
Inertance	M	$\frac{1}{m}$	Inductance	L	Henries
Compliance	C	ms^2	Capacitance	C	Farads

The analogs in this table can be used to establish element models for different sections of the muffler system. In a later section, a few specific examples of acoustic element models will be discussed in greater detail.

Once the models for the various elements in a muffler system are defined, the performance of the system can be found using the transfer matrix method (as defined in [1]). The transfer matrix method involves representing each passive element in the muffler system as a two port element, and then finding a 2x2 matrix, known as a transfer matrix, which describes the interaction between elements. The transfer matrices describe the relationship between pressure and mass velocity through each of the ports. An example of a transfer matrix is given below.

$$\begin{bmatrix} p_n \\ v_n \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} p_{n-1} \\ p_{n-1} \end{bmatrix} \quad (1)$$

For solving a general muffler the steps of the transfer matrix method are as follows:

1. Draw the equivalent circuit diagram for the system
2. Write down the transfer matrices for each element
3. Multiply the matrices sequentially

Drawing the Circuit Diagram

In a muffler system, the circuit diagram is usually composed of a source and source impedance, an arbitrary length chain of elements, and a radiation impedance.

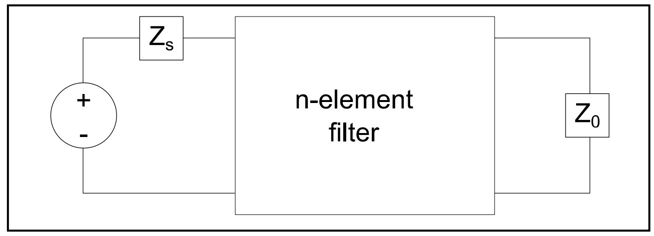

Figure 1: Generic circuit diagram for a muffler system.

Within this circuit diagram, the n-element filter 2 port element encapsulates an arbitrary number of 2 port filters, which need to be constructed from their equivalent models.

The most common types of two port elements are distributed, shunt, and series elements. Distributed elements are relatively long compared to the wavelengths being analyzed. This type of element is usually used to represent the piping in the muffler system. Shunt elements are elements in which the pressure field is uniform across them, but which allow mass flow to be diverted. Series elements have a constant mass flow across the ports, but cause a pressure drop. An example of the different circuit schematics for each element type is given below.

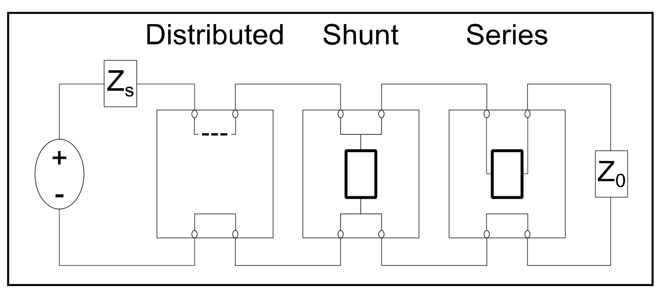

Figure 2: An example of an element chain composed of the three different most common types of elements.

Determine the Transfer Matrices

The transfer matrix of an element can be found in two steps. First, the impedance of the element needs to be determined. This is usually derived from first principals. Next, the configuration of the impedance element needs to be determined. This will either be ‘distributed’ ‘shunt’ or ‘series’. Once the correct configuration is selected, the impedance is substituted into the appropriate fields of the matrix.

A distributed element transfer matrix will have the following form:

$$\begin{bmatrix} \cos(k_0\, l) & (\text{j Y})\sin(k_0 l) \\ (j/y)\sin(k_0 l) & \cos(k_0 l) \end{bmatrix} \quad (2)$$

A lumped series element:

$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \quad (3)$$

A lumped shunt element:

$$\begin{bmatrix} 1 & 0 \\ 1/Z & 1 \end{bmatrix} \quad (4)$$

Solving the System

Once each transfer matrix is found, they can be combined and used to calculate the pressure and mass flow after the element from the pressure and mass flow prior to the element, as shown in equation (1). By substitution using equation (1) it is evident that a chain of multiple matrices can be simplified into a single 2x2 matrix by taking the matrix product of each transfer matrix in turn. The final matrix gives no information about the pressures and mass flows within the element chain, but greatly reduces the number of calculations necessary to compute the full frequency response of an entire muffler system.

After an element chain has been reduced to a single transfer matrix, different pieces of information can be extracted. If we let T be the total transfer matrix, Y_1 be the characteristic impedance of the input port, and Y_2 be the characteristic impedance of the output port, then the transfer loss of the muffler system can be found using the equation given below.

$$TL = 20\log\left[\left(\frac{Y_1}{Y_2}\right)^{1/2}\left|\frac{T_{11} + \frac{T_{12}}{Y_1} + Y_2 T_{21} + (Y_2/Y_1)T_{22}}{2}\right|\right] \quad (5)$$

The insertion loss can be found using a similar formula.

$$IL = 20\log\left[\frac{Y_1}{Y_2 + Y_1}\left|T_{11} + \frac{T_{12}}{Y_1} + T_{21}Y_2 + \frac{Y_2}{Y_1}T_{22}\right|\right] \quad (6)$$

Note that in the case where $Y_1 = Y_2$ that equations (5) and (6) simplify into the same equation, meaning that in this case $IL = TL$.

Estimating Backpressure

Similar to calculating pressure drop across a muffler system, the back pressure created by an element may be estimated using the transfer matrix method.

Transfer matrix elements must be set up using a given flow resistance for the impedance of the element rather than the acoustic impedance. Like the acoustic impedance, the flow resistance may be calculated using first principals.

Once the transfer matrices have been calculated, the same method as used to estimate the acoustic performance of a muffler system may be used to calculate the flow resistance.

See [2] for more information in calculating the backpressure of a system.

Helmholtz Tuner Lumped Element Model

Helmholtz tuners are very useful filter elements in exhaust systems because they provide a very high attenuation at a specific frequency with almost no added restriction. As frequencies pass into the Helmholtz cavity, they set up a resonance in the chamber which produces a wave at a 180 degrees phase offset from the incident wave. The wave splits at the duct, and travels downstream and upstream. The downstream wave destructively interferes with the incident wave, producing a net canceling effect. However, the upstream traveling wave constructively interferes with the incident wave. This effectively amounts to reflecting the wave energy back at the source, where it is absorbed as heat energy.

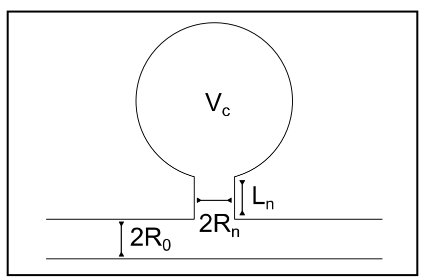

Figure 3: A schematic equivalent for a Helmholtz resonator

An example of a Helmholtz tuner schematic is given in figure 3. The tuner is composed of a neck and a volume. The air in the neck acts as a mass, providing inertia to the system. The air in the volume compresses and expands as the air in the neck presses upon it, which is analogous to a spring. Together, these components act like a second order mass spring dampened system, and an acoustic impedance can be derived as such.

The acoustic impedance of the branch going into the Helmholtz resonator can be written as follows. Let S be the cross sectional area of the neck, L be the length of the neck, V_c be the volume of the cavity, c be the speed of sound in the transmission medium, and j be the imaginary unit $\sqrt{-1}$.

$$Z = j\,\omega \frac{L}{S} - j\frac{c^2}{\omega\, V_c} + \frac{\omega^2}{\pi\, c} \tag{7}$$

The first term in this equation represents the inertance of the mass within the neck. The second term represents the compliance of the volume, and the third term represents the radiation resistance on either side of the neck tube [1].

Studying the schematic in Figure 3, it is evident that the pressure before and after the neck must be continuous. However the mass flow splits at the neck. This indicates that the Helmholtz resonator acts like a shunt element. Therefore, equation (7) may be substituted into equation (4) to produce the transfer matrix for a Helmholtz element.

Expansion Chamber Lumped Element Model

The expansion chamber is a good contrasting element to the Helmholtz chamber. While the peak reduction of an expansion chamber is lower than a Helmholtz tuner, this element produces a much more broadband response. Like the Helmholtz tuner, this element can provide low flow restrictions, although in general, the flow restriction increases with the peak attenuation level.

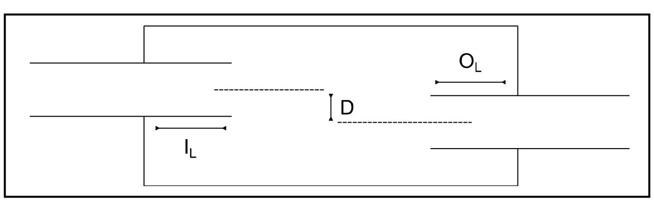

Figure 4: Schematic example of an expansion chamber

An expansion chamber is composed of an inlet, an outlet, and a chamber which is wider than either of the ports. Within an expansion chamber, the size of the chamber, the amount of overhang of the inlet and outlet, and the offset distance between the inlet and outlet all significantly change the acoustic impedance and flow.

An expansion chamber can be split into two different resonators, each which can be divided into three sections with three impedances. At the interface of each section and between the resonator, mass flow and pressure field continuity must hold.

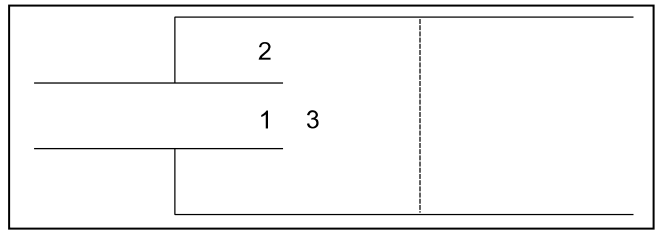

Figure 5: Three impedances which must be taken under consideration in an expansion chamber.

Within the chamber, sections 1 and 3 act as distributed elements. Section 2 must be given special consideration. The impedance at the interface for section 2 is given in the following equation, where R_{end} is the reflection coefficient at the plate end of section 2, Y_2 is the cross sectional area of the expansion chamber, k_0 is the wavenumber, and l_2 is either I_L or O_L depending on which side is being analyzed [1].

$$Z_2 = Y_2 \frac{1 + R_{end}\, e^{-2jk_0 l_2}}{1 - R_{end}\, e^{-2jk_0 l_2}} \tag{8}$$

Note that as R_{end} approaches 1 (for a stiff plate) the following approximation becomes accurate.

$$\lim_{R_{end} \to 1} Z_2 = -j\, Y_2 \cot(k_0\, l_2) \tag{9}$$

In a branch resonator, the pressures at the interface of 1,2,and 3 must be equal, where as the mass flow may split between either 1,2, or 3. Therefore, similar to a Helmholtz resonator, the Z_2 impedance of an expansion chamber acts like a shunt element between the distributed elements which represent sections 1 and 3.

Note that with this technique, it is not possible to analyze expansion chambers whose inlet and outlets are offset from the center of the chamber. To perform that analysis, a more powerful technique must be used, which is discussed in [1].

OPTIMIZING ELEMENT CHAINS

To be an effective product, the modular software needs to severely reduce the engineering time spent designing a muffler solution. The goal of the system is to automate the procedure enough allow an acoustics layperson to develop a full muffler solution without the aid of a NVH engineer. This is accomplished by reducing the design interaction process to entering readily available and widely understood specifications, and using numerical iteration to arrive at an appropriate solution.

The first specification required by the program is the available space claim for the muffler. It is assumed that the available space claim will be some form of an extruded shape. Therefore, the user is asked to choose a cross-sectional shape, and set some basic dimensions, and give a maximum length.

Next, the user needs to provide a target insertion loss curve, or an auto-power spectrum characterization of the engine without muffler and target spectrum. If the user has the target insertion loss, this step is simple. If the user must characterize the engine, then a small amount of engineering time may need to be used to collect the engine's auto-power spectrum. The user will also need to specify a maximum static backpressure. This information should be enough for the computer to develop a reasonable acoustic model solution.

Currently, we think it will be possible to reduce the space of possible muffler solutions enough that a brute force iterative method can be used to solve the system. The goal is to reduce the possible space of solutions by restricting the total number of potential elements to a reasonable number, and constraining the step size and bounds on the parameters of each element. One of the significant advantages of using transfer matrices to do the design is that a full solution can be computed on a normal processor within microseconds.

A point of consideration in developing the acoustic model is that eventually a manufacturing model will need to be developed from the solution. This means that care needs to be taken so that the algorithm does not find un-manufacturable solutions. Currently, it is anticipated that simple conservative geometric rules will need to be programmed into the system which take into account the entered space claim. However, because it will be difficult to prove that the constraints hold in all possible cases, a collision detection algorithm should be run when the manufacturing model is generated. If an interference is detected, another solution will need to be found.

If the design space becomes too large for a brute force method, more advanced global optimization algorithms may be employed. An optimization method such as the metropolis algorithm or simulated annealing may be employed. [3]

Once a valid solution is found, a manufacturing model must be created from it.

GENERATING MANUFACTURING MODELS

The manufacturing models contain a physical realization of the purely conceptual acoustic models. Within the generation of the manufacturing models, a physical solution needs to be found which packs the elements described by the acoustic model into the give space claim.

The current plan is to use parametric solid models generated from SolidWorks as the basis of the manufacturing models. Parametric elements will be generated which are easily modifiable and easy to mate together. The SolidWorks API will then be used to set the parameters in each element accordingly and mate all of the elements together into a full muffler system.

To date, a parametric model containing three fixed elements has been built. The elements contained are a Helmholtz resonator, an expansion chamber, and a perforated resonator. The model is driven by an assembly level sketch, and is fully configurable.

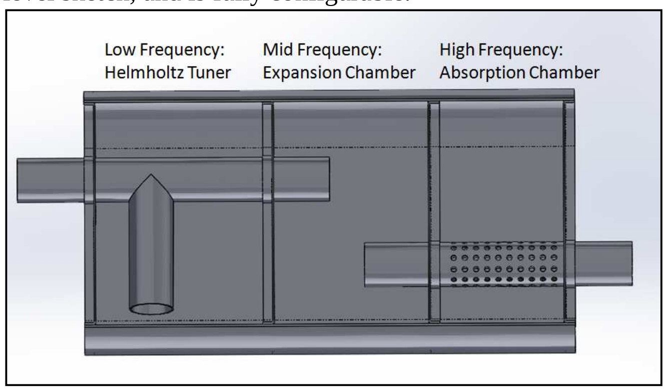

Figure 6: Parametric muffler concept. Not shown is the driving sketch which allows each chamber to be customized.

TECHNICAL DATA PACKAGE GENERATION

The technical data package (TDP) consists of all the information which is needed for the manufacturing job. It will be in the form of a directory hierarchy, and distributed as a compressed archive.

The technical data package will contain the following pieces of information:

- Manufacturing Drawings
- Code for running CNC machinery
- Documentation regarding acoustic chain

- Logistics information for resource management

MANUFACTURING PROCESSES

Existing low-production muffler manufacturing technology are being adapted to meet modular exhaust design needs. Parametrically limited muffler designs that are established in this effort will be used to develop meaningful lists of muffler components for specific or broad applications. These component lists provide data necessary to optimize the shop layout plan, schedule, selection and use of machinery/tooling, and the handling of inventory and materials.

Manufacturability will be maintained throughout the expansive design space by limitations imposed onto the parametric SolidWorks manufacturing model. A “variable cross-section” design concept has been chosen as the best suited design for optimal modularity. This imposes limitations to the cross-sectional shape of the muffler, while providing variability in cross-sectional size. A large amount of variability is also available for baffle placements and other internal acoustic features used to tune the muffler to a specific application.

Processes identified and developed for modular muffler manufacturing include the following: Roll forming, brake, seam welding, spot welding, CNC cutting, CNC roll, Seam welding, tube bending, stamping. Assembly processes include the following: Modular fixture table, pipe expander.

Cost and market studies will determine the level of automation to be included into the modular process. Material selection is a key player in reducing cost. Material studies combined with design and analysis produce products that are optimized for cost.

COST AND LEAD TIME ESTIMATES

Costs and lead times for modular mufflers will be significantly reduced due to several important factors. The complicated shapes, manufacturing challenges, and large amount of engineering required of custom exhaust systems has historically driven the high costs and lead time. With modular design and manufacturing, GLSV is able to direct the design efforts into simpler shapes while retaining the ability to customize the system to the customer’s dimensional needs. Acoustic and flow performance will not only be maintained, but in many cases will be surpassed due to the integration of the optimization software currently in development. Automation of technical data generation, optimized plant layout, and customized tooling/machinery will ensure the manufacturing will perform effectively for the targeted market size.

A cost per muffler unit of $500 for a volume of 100 units was established as a loose goal for the modular effort (obviously cost will vary heavily based on size and application). This low cost is complimented by lead times approaching two weeks. Achieving this goal without sacrificing performance would be a large accomplishment in the low production muffler market. This new process would not only reduce costs for the customer procuring the mufflers, it would also save costs associated with maintaining and storing inventory.

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